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*CCSS Mathematical Practice 4: Model with Mathematics

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Editor's Note: Because at times the tables and figures were too large to nicely embed within the text, or references in the article came too close together to fit the graphic, the decision was made to use a hyperlink to enable you to toggle back and forth between text and table or figure. Clicking on the underlined table or figure within the text will take you to the referenced table or figure. Clicking on the underlined matching table/figure will bring you back to the text. The tables & figures are located at the end of the article.

The Common Core State Standards (CCSS) specify a learning path of mathematical topics that begin in Kindergarten and build in each grade in a coherent and deep way. Students are to understand and explain mathematical concepts and operations, and students are to become fluent with crucial mathematical competencies. Eight mathematical practices (MP) are also an important part of the CCSS; these describe important aspects of how students should function mathematically.

In this paper I will first overview a framework for understanding how teaching can move from understanding to fluency with the support of all of the mathematical practices. Then I will focus on the important Mathematical Practice 4: Model with mathematics. Examples of how students model with mathematics for the crucial domains of NBT (Number and Operations in Base Ten) and OA (Operations and Algebraic Thinking) will be discussed. How teachers can support such modeling in the classroom will be overviewed. This paper focuses on Grades K to 5 because it is crucial that modeling gets off to a great start in these grades. How modeling changes for older students is briefly discussed at the end.

The Classroom as a Math Talk Community

Recommendations of earlier national reports on which the CCSS depend can be viewed within the three phases of classroom teaching shown in Figure 1. All reports including the CCSS emphasize the importance of a classroom functioning as a math talk community in which students make sense of mathematics and discuss and explain their own mathematical thinking. Coherent visual learning supports are vital for the functioning of the math talk community. Description of a math talk community and advice from teachers about how to build a math talk community are summarized in Hufferd-Ackles, Fuson, & Sherin (2014), and in Fuson, Atler, Roedel, & Zaccariello (2009). Aspects of such a community are described in a webcast where examples of math talk are given (Math Talk Community Part 1), and examples and discussion of building and extending such a community are provided in Part 2 of this webcast (Math Talk Part II).

For each new mathematical topic, a teacher begins by eliciting and discussing student methods (Phase 1). The teacher then rapidly moves on to deeper discussions in which research-based mathematically-desirable and accessible methods are elicited from or made available to students. Students move from their more primitive methods to understanding and explaining these Phase 2 methods.

The eight mathematical practices were formed into four pairs and given names. These pairs and names are shown in Figure 2. These pairs enable the CCSS teaching task to be summarized into a single sentence: *Today did I do math sense-making about math structure using math drawings to support math explaining? Can I do a bit better tomorrow?* Use of these mathematical practices within the math talk community enables students to understand, explain, and eventually become fluent with a compact method.

Mathematical Practice 4: Model with Mathematics:
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two- way tables,

graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose. (CCSS, p. 7)

In Table 1 you can see the content standards from each grade that show how MP4 also appears within specific standards. These examples are from the major problem solving domain, OA:Operations and Algebraic Thinking, and from the two major numerical domains, NBT: Number and Operations in Base Ten and NF: Number and Operations-Fractions. Other examples appear in the standards for these and the other domains. These exemplify how central MP4 is in mathematical functioning.

Modeling with mathematics requires students to *mathematize* a situation: To focus on the specific aspects that are mathematical. Students then represent the situation with a math drawing and/or an equation or other mathematical form that highlights the mathematical relationships involved. This modeling requires moving back and forth between the situation and the mathematical representation, modeling that situation in order to ensure that the mathematical model makes sense.

Math drawings or diagrams are crucially important math models in grades K to 5. They continue to be summarized in Table 2.

MP 4 Modeling for Multi-digit Computations (NBT)

In the CCSS NBT standards, learning to add, subtract, multiply, and divide multi-digit numbers is a sense-making endeavor in which students develop and explain written numerical methods connected to drawings or objects. The visual models help students focus on the quantities involved (hundreds, tens, and ones) and understand the written numerical methods as involving different multi-unit quantities, rather than just the single digits that students write. In Figure 3 research-based visual models I used in classrooms for more than ten years are shown on the left followed by written methods for each operation. The written methods in the middle are Phase 2 methods (see Figure 1 for the three phases) that are accessible to students and are mathematically desirable. Current Common methods are shown on the right. These methods are sometimes called standard algorithms, but in fact there is no agreed upon single standard algorithm in this country. There are written variations of the standard algorithmic approach (see the discussion in Fuson & Beckmann, 2012/2013).

Students use MP4—they model with mathematics—when they make math drawings like those shown in Figure 3 or make other visual models that show the place value meanings of the digits as quantities of hundreds, tens, and ones. Figure 4 shows how useful MP4 is to a student explaining her method to her classmates. Such mathematical visual models also help her classmates understand her explanation and her written method. In step a, the math drawings show the hundreds, tens, and ones and support students to see that they need to add like quantities to each other: hundreds to hundreds, tens to tens, and ones to ones. In steps b and c the math drawings help students see and understand composing ten of a unit to make one of the next larger unit to the left. With these visual models, students draw quantities in 5-groups that allow them to see easily how many more they need to make ten. This supports the mental method of making a ten that can be used when students become fluent and no longer draw the models: 9 + 7 is thought of as "7 gives 1 to the 9 to make ten, leaving 6 to go with the ten, so 16." Step d is the final step of adding the hundreds. The written method shows three 1s in the hundreds column to add, but the visual model makes clear that two of these 1s come from the original hundreds quantities and the bottom 1 comes from the composed ten tens.

After an explanation, it is helpful to ask if anyone has any questions. Here we can see from the questions that the explainer's classmates paid close attention to her explanation, and they looked at how the drawing was related to the written method.

When students just begin to participate in a math talk community, the teacher will need to model and extend student explanations. But many students can explain in detailed ways such as you see here. Other students can help the explaining be clear with questions or suggested edits of explanations. It is helpful for teachers to stand at the side or back of the room so that the explainer will look at classmates while also looking at the teacher. Teachers need to wait while students think and not jump in and explain for the student. My teachers call this "biting your tongue" and they find that it is very difficult to wait for the math talk to emerge. But it will emerge if the classroom feels safe and classmates help each other explain. We can see how all of the mathematical practices work together to support each of them. Students improve the math modeling as they explain their reasoning and listen to and make sense of math models of other students.

MP 4 Modeling for Operations and Algebraic Thinking (OA)

The OA standards include the learning paths for single-digit addition/subtraction in Kindergarten through Grade 2 and for multiplication/division in Grade 3. We already saw MP 4 in action for OA in the math drawing, explanation, and questions when adding 9 ones and 7 ones to make 16 ones and when adding 5 tens and

8 tens to make 13 tens. Children move through three conceptual levels in adding. They initially count all of the things for both addends, such as counting from 1 to 9 ones and then continuing the count with the 7 ones as 10 to 16. But early in Grade 1 children can conceptually embed the addends within the total, so they can see the 9 ones and 7 ones drawn there as embedded within the 16 total ones. So they can begin the final total count by pretending they have already counted the 9 ones and then count on the 7 ones. So the explainer could have found her answer and said, "I counted on from 9 seven more: **9**, 10, 11, 12, 13, 14, 15, 16, pointing to each of the 7 ones as she counted them within the final count of the total or keeping track with fingers or with a rhythm." But this is a Grade 2 addition problem, so many students add with the level three make-a-ten method of recomposing the given addends into an easier problem: 9 + 7 becomes 10 + 6. So we can see that MP 4 operates for these single-digit operation aspects of the OA standards much as it does for the NBT standards for multi-digit operations because what students are modeling is the units in the quantities.

The other major content for the OA standards is the kinds of situations that give meaning to the operations adding, subtracting, multiplying, and dividing, regardless of the nature of the quantities involved (e.g., multi-digit numbers, fractions, decimals, measures). These situations are specified by name within the standards and are summarized in Tables 1 and 2 on pages 88 and 89 of the standards document (Common Core State Standards Initiative, 2010). Each of the three types of addition-subtraction problems and each of the three types of multiplication-division problems involve three quantities. Each of these quantities can be the unknown. The types differ in how the three quantities are related to each other in the situation in the real world.

Math drawings for all of these situations are shown in Figure 5 circle orange. These math models of the situations give a sense of each problem type. Addition-subtraction types are shown across the top, and multiplication-division types are across the bottom. The grade level at which students begin work with a given type is shown at the top right of each drawing. Students begin with the easiest unknown and progress toward the more difficult unknowns and difficult problem language. Students can make their own math models for the situation. But I found that as numbers got larger, students drew all of the quantities. They instead needed help mathematizing the situation to focus on the relationships among the quantities. The math drawings shown in Figure 5 are the result of ten years of research in classrooms identifying and testing out the best drawing for each problem type. These math drawings can be used for any quantities: small numbers, multi-digit numbers, fractions, decimals, or measures. So they provide continuity across the grades as students think about, represent, and solve increasingly difficult problem situations.

Let us now look at students engaging in MP 4 for a difficult type of addition problem, Add To Start Unknown. The problem and the math models of three second-grade students are given in Figure 6. The student at the top left has written a situation equation, labeled the parts of the equation to relate them to the parts of the situation, and has also drawn quantities to the right. We call this a situation equation because it shows the mathematical relations in the situation: We don't know how many Yolanda had to start, so a mystery box with no number in it is written first. Eddie took 7, so – 7 is written next. We know that Yolanda had 5 left at the end, so the equation finishes with = 5. The student can see from the equation that the beginning total has been broken apart to make 7 and 5. So adding 7 and 5 will make that beginning total. These students are used to making their math models using groups of 5, just as in the drawings we saw for the 3-digit addition problem. The student drew 5 circles and then 7 circles, so he is moving backwards in the equation to make the beginning total. The answer 12 is written in the equation and on the problem answer space.

The second student math model shown on the top right uses the drawing at the top middle of Figure 5 to show that the 7 and 5 are put back together. The student also wrote this in words below the drawing to explain the model. We would need to watch the student explanation to see how the student actually added the 7 and 5 to get 12.

The third student did not represent the situation but instead wrote a solution computation. She represented the situation mentally, realized that she had to add the two addends to make the total, and wrote that as a computation. She did label her computation to relate it to the situation.

Together these math models show us a lot about how students represent and solve word problems. Students begin by understanding the situation and then mathematize the situation to focus on the mathematical relationships. Many students naturally write situation equations to represent the problems with the more difficult unknowns. Or they might write a situation computation to represent the situation, such as a vertical subtraction with a mystery box at the top, -7 below, and 5 as the answer. Or they might write a break-apart drawing to show the total-addend-addend relationships. Once students have represented the situation, they look at their situation representation to see how to solve it to find the answer. For this, it is important that they understand which quantity is the total and which quantities are the addends. Students use their understandings about these relationships to decide which operation to carry out.

What I have just described is *algebraic problem solving*. To solve an algebra problem, we first understand and represent the situation, often with an equation. Then we use algebraic techniques to solve for the answer. We see that the situated problem solving students can experience with the problem types in Figure 5 (those in the CCSS) allows them to solve equations without knowing the algebraic techniques. We also see that students represent and solve a given problem in different ways. Seeing the math models drawn and labeled and explained by classmates helps students make more relationships among the situation and various math models. It helps students make equations meaningful and understand where the total and addends are in various math models including equations.

One can see the problem solving phases more clearly with difficult unknowns such as start unknown. But even with the simplest problem situations solved by Kindergarten children, students first represent the situation and then find the solution. An example Add To Result Unknown problem is *3 dogs were barking. Then 2 more dogs started barking. How many dogs are barking in all?* Children may draw 3 circles and then draw 2 more circles. This represents the situation. It is now simple to answer the question because all of the dogs are represented in the math model, and they just need to be counted. But the same problem solving steps exist from the beginning of problem. Children need to be helped to focus on the mathematical aspects of the situation. They may need help understanding the problem text. Teachers can support understanding with actions such as the following:

- Act out the situation.
- Say it in your own words.
- Tell the situation in different words.
- Ask the same question but use different math words (e.g., in all, altogether, the total).
- Tell the same situation but about different things.

Children can be successful problem solvers if they use one major strategy: Understand the situation, and make a math model to help. The math model can be a math drawing, an equation, or both. Labeling the math model to connect it to the situation is very helpful.

The addition and multiplication comparison situations shown on the right of Figure 5 have particularly complex language that needs to be learned and practiced. The multiplication-division problem types are actually easier for many students than the more difficult unknowns in the addition-subtraction problem types. The main difficulties are with learning all of the single-digit multiplications and divisions (see Sherin & Fuson, in press, for an overview of such learning). Some 2-step and 3-step problems can be represented with one equation or drawn math model. But some problems are clearer with two equations or two or more drawn math models.

MP 4 Modeling for Other Math Domains and for Older Students

Geometry uses visual models that are different from the quantity and situational models we have been discussing. Seeing accurate 2D and 3D models of geometric shapes is necessary for students to decompose shapes into their component parts to understand and categorize them. Students can sketch math models to help them in geometric problem solving, but they also learn how to use tools to make more accurate representations of geometric shapes and situations.

Measurement and data modeling involve learning to focus on the measure attributes and relationships and to use measurement tools and data display tools. Understanding length units is crucial for much of this modeling. Students are often misled by the numbers on rulers and graph scales because these are at the end of length units but look as if they are just labeling the point at which they occur. Having students slide a finger along each length unit as it is counted, or drawing long ovals around each length unit, can help students understand, use, and make these models accurately.

The meaningful bases for much of later mathematics can be built in grades K to 5. Later topics involve more complex or abstract problems, relating different math concepts, and using technological tools to model these more-advanced problems. But the fundamental approach to mathematics teaching and learning is still captured by our sentence about the four pairs of mathematical practices: *Today did I do math sense-making about math structure using math drawings to support math explaining? Can I do a bit better tomorrow?*

Math drawings might now be done on a graphing calculator, but hand sketches as a basis for math modeling and thinking still play a role. If teachers support well the central role of MP 4 Model with Mathematics within the eight mathematical practices in the early grades, students can be capable math modelers all of their lives.

Additional Resources

I developed over 10 hours of webcasts to explain the CCSS-Math and show visual supports for student understanding. The examples come from my research-based Kindergarten to Grade 6 math program *Math*

Expressions published by Houghton Mifflin Harcourt, but these webcasts can be used by teachers using any math program. Visual supports are very important in the CCSS-Math, so these webcasts can help teachers and other educators or parents understand the CCSS-Math and how students can be learning in the classroom. You can move around within a webcast by clicking on the slide titles on the left. MP 4 model with mathematics is used frequently in these webcasts to explain CCSS-Math concepts. The links to these webcasts are listed under Projects on my Northwestern University webpage: http://www.sesp.northwestern.edu/profile/?p=61

The following books, developed for teachers by NCTM, are also helpful about using math models. Their titles refer to the focal point document, but they are consistent with the CCSS which extended the focal point document:

- National Council of Teachers of Mathematics (NCTM) (2009). *Focus in Grade 5: Teaching with Curriculum Focal Points*. Reston, VA: NCTM.
- National Council of Teachers of Mathematics (NCTM) (2010). *Focus in Kindergarten: Teaching with Curriculum Focal Points*. Reston, VA: NCTM.
- National Council of Teachers of Mathematics (NCTM) (2010). *Focus in Grade 1: Teaching with Curriculum Focal Points*. Reston, VA: NCTM.
- National Council of Teachers of Mathematics (NCTM) (2011). *Focus in Grade 2: Teaching with Curriculum Focal Points*. Reston, VA: NCTM.

References

Common Core State Standards Initiative (2010). *Common core state standards for mathematics*. Retrieved from http://www.corestandards.org/Math

Fuson, K. C., Atler, T., Roedel, S., & Zaccariello, J. (2009). Building a nurturing, visual, Math-Talk teaching-learning community to support learning by English Language Learners and students from backgrounds of poverty. New England Mathematics Journal, (May) XLI, 6-16.

Fuson, K. C. & Beckmann, S. (Fall/Winter, 2012-2013). Standard algorithms in the Common Core State Standards. *National Council of Supervisors of Mathematics Journal of Mathematics Education Leadership, 14* (2), 14-30.

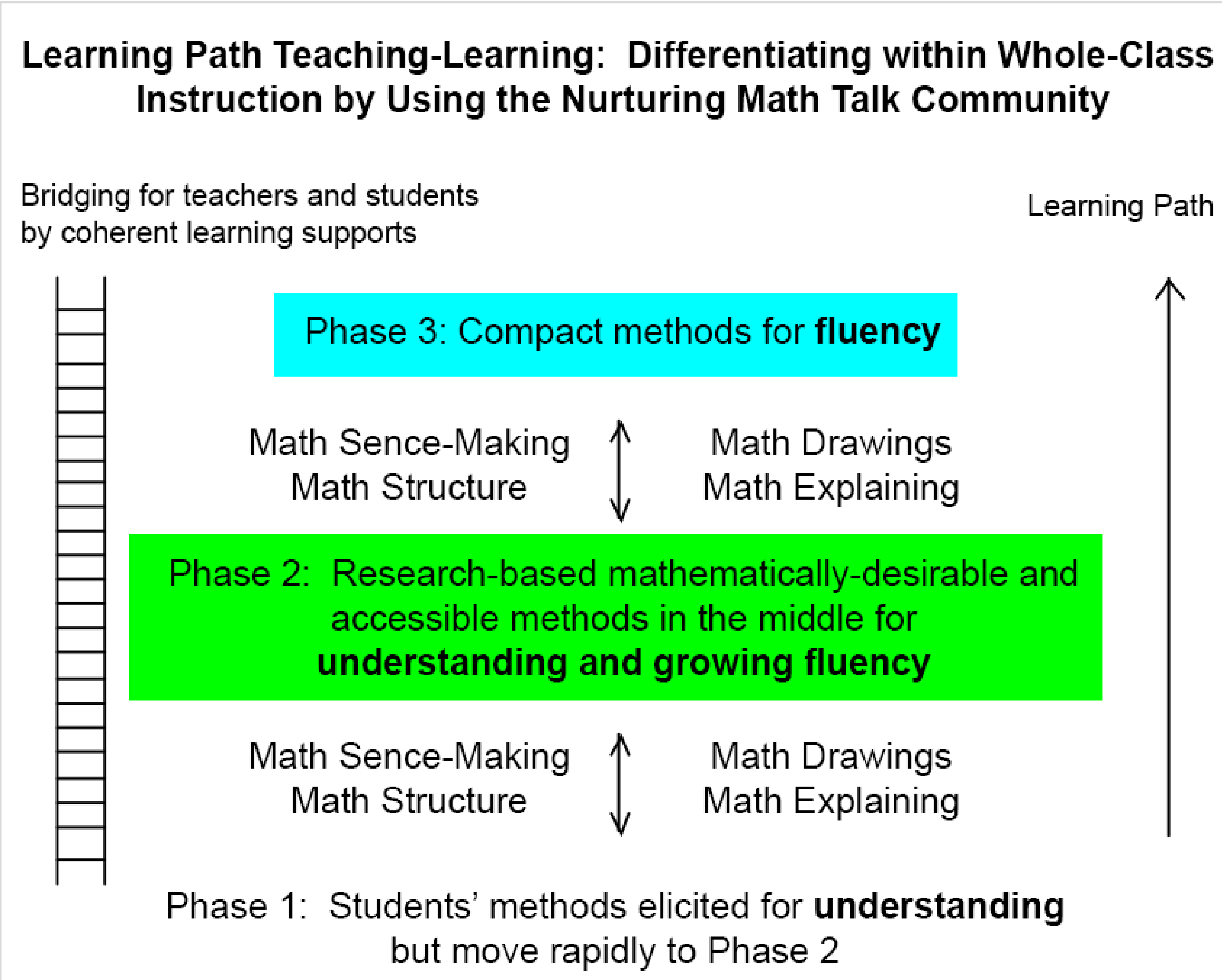
Hufferd-Ackles, K., Fuson, K. C., & Sherin, M. (2014). Describing levels and components of a Math-Talk Learning Community. In E. A. Silver & P. A. Kenney (Eds.), *Lessons Learned from Research: Volume 1: Useful and Useable Research Related to Core Mathematical Practices*. Reston, VA: NCTM.

Murata, A. & Fuson, K. C.(in press). Class Learning Zone and Class Learning Paths: Responsive Teaching in First-Grade Mathematics. In E. A. Silver & P. A. Kenney (Eds.), *Lessons Learned from Research: Volume 2: Useful and Useable Research Related to Core Mathematical Practices*. Reston, VA: NCTM.

Sherin, B. Fuson, K. C. (in press). Multiplication methods in the context of the Common Core State Standards. In E. A. Silver & P. A. Kenney (Eds.), *Lessons Learned from Research: Volume 2: Useful and Useable Research Related to Core Mathematical Practices*. Reston, VA: NCTM.

TABLE & FIGURES

FIGURE 1: *Three Phases of Classroom Teaching*



This figure is an extension of Fuson, K. C. & Murata, A. (2007). Integrating NRC principles and the NCTM Process Standards to form a Class Learning Path Model that individualizes within whole-class activities. National Council of Supervisors of Mathematics Journal of Mathematics Education Leadership, 10 (1), 72-91. This model summarizes several National Research Council Reports.

FIGURE 2: *Pairing the Practice Standards to Better Summarize the Teaching Tasks.*

Common Core Mathematics Practices

Math Sense-Making about Math Structure using Math Drawings to support Math Explaining Teachers observe daily student use of these mathematical practices of desirable behavior.

Math Sense-Making: Making sense and using appropriate precision
1 Make sense of problems and persevere in solving them.
6 Attend to precision.

Math Structure: Seeing structure and generalizing
7 Look for and make use of structure.
8 Look for and express regularity in repeated reasoning.

Math Drawings: Modeling and using tools
4 Model with mathematics.
5 Use appropriate tools strategically.

Math Explaining: Reasoning and explaining
2 Reason abstractly and quantitatively.
3 Construct viable arguments and critique the reasoning of others.

Table 1. Model with mathematics appears in many central standards

Here is one example from each grade level:

K.OA.2. Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.

1.OA.11. Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

2.NBT.7. Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method.

3.OA. 3. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

4.NBT. 5. Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

5.NF. 4. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. a. Interpret the product (a/b) x q as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a x q ÷ b. For example, use a visual fraction model to show (2/3)x 4 = 8/3, and create a story context for this equation. Do the same with (2/3) x (4/5) = 8/15. (In general, (a/b) x (c/d) = ac/bd.)

Table 2. Advantages of Student Math Drawings of Problem Situations and/or Solution Methods

Math drawings are simplified drawings that show **quantities** (e.g., 238 or 6 dogs) and **relationships** in simple ways (e.g., drawing 2 hundreds-squares, 3 tens-sticks, 8 circles for 238 or drawing 6 circles for the 6 dogs).

- Young children aged 2 through kindergarten benefit from using **physical objects** to show mathematical ideas. But they also need experience with 2-D representations on paper (like math drawings) to help understanding pictures and drawings in books.
- **Manipulatives** are important for introducing some math ideas for older students (e.g., making fraction strips by folding unit fractions).
- But many ideas beginning in grade 1 benefit from having **students make math drawings** and relate the drawings to the formal math symbols.

Many students **take pride** in their math drawing creations and **use care** in making and in editing their drawings. They **have a product** at the end of their solution.

Student math drawings of problem situations and/or solution methods can be **made or reflected onto the classroom board** or wall so that **everyone can see them**. They enable students to explain their thinking more clearly and explicitly

- **by pointing to parts** of their drawing as they explain;
- **by labeling parts** of the drawing to relate those parts to the problem situation;
- **by using an arrow or other means of relating a step** with quantity drawings (e.g., making 1 new ten from ten ones) to that **same step in the numerical method** (e.g., writing the new 1 ten in the tens column).

These aspects **enable listeners to understand** because of the **relating of hearing and seeing**.

Math drawings are **windows into the minds** of students that allow teachers to understand student approaches and errors on homework and classwork. They enable teachers to do **continual assessment for instruction**. Teachers can always follow up on a math drawing by asking a student to explain it, but this frequently is not even necessary to **understand student thinking**.

Math drawings **remain after the problem is solved** and can be a **record of the whole action**, whereas actions with manipulatives may be over by the time the teacher gets to a particular group or only show the current step. The **teacher can collect math drawings** made on paper after class to **reflect on student methods** shown that day. This cannot happen with manipulatives.

Math drawings are **easier to manage** than are manipulatives. They are not dropped on the floor, thrown at other students, lost, mixed up, taken from the school by last year's teacher, or lost during summer school.

FIGURE 3: *Written Variations of the Standard Algorithms*

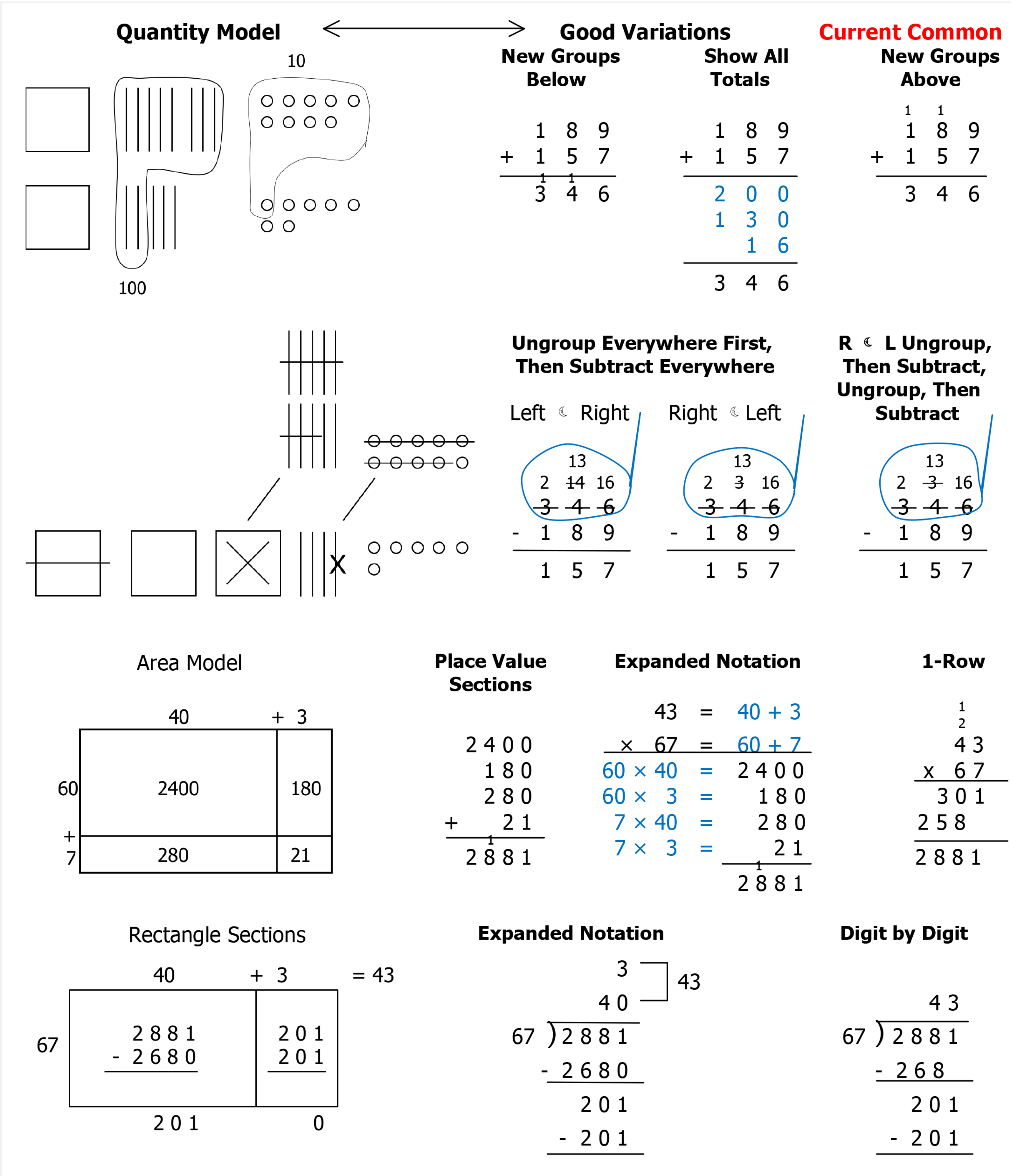
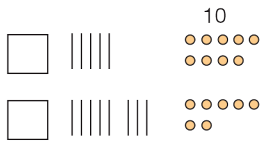
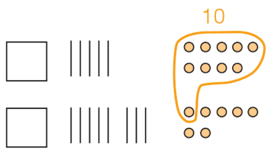
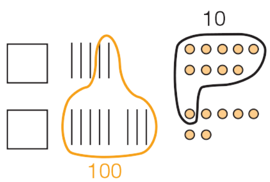
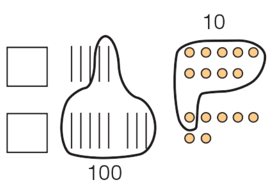


FIGURE 4: *Using Visual Models to Explain One's Thinking*

Math Drawing and Problem	Explanation Using Place-Value Language About Hundreds, Tens, and Ones
a.  159 + 187	I drew one hundred, five tens, and nine ones to show one hundred fifty nine, and here below it I drew one hundred, eight tens, and seven ones for one hundred eighty seven. I put the ones below the ones, the tens below the tens, and the hundreds below the hundreds so I could add them easily.
b.  159 + 187 6	See here in my drawing, nine ones need one more one from the seven to make ten ones that I circled here, and I wrote 10. That leaves six ones here. With the numbers the seven gives one to the nine to make ten that I write over here in the tens column, see one ten. And I write six ones here in the ones column.
c.  159 + 187 46	With the tens, I start with eight because it is more than five so it is easier. I get two tens from five tens to make ten tens, see here, and I write one hundred here to remind me that the ten tens make one hundred. There are three tens left in the five tens and I have one more ten from my ones (see here in my drawing and the one ten at the bottom of the tens column). That makes four tens and the one hundred. So in my problem I write the one hundred below in the hundreds column and the four tens in the tens column.
d.  159 + 187 346	There are three hundreds, two in the original numbers I'm adding and one new hundred from the ten tens. I write three hundreds here in the hundreds column. Are there any questions? Yes, Stephanie.

Student Question	Explainer Answer
Stephanie: For the tens, you never said fourteen tens as the total of the tens. Why not?	Because when I'm making ten tens, I just can write that one hundred over here with the hundreds and just think about how many tens I need to write. But I can think eight tens and five tens is thirteen tens and one more ten is fourteen tens, so that is one hundred and four tens. You can do it either way. (Aki)
Aki: Do you still need to make the drawings or did you just make them so you could explain better?	I don't have to make the drawings, but I can explain better with a drawing because you can see the hundreds, tens, and ones so well. (Jorge)
Jorge: Do you do make-a-ten in your head or just know those answers?	I just know all of the nine totals because of the pattern: the ones number in the teen number is one less that the number added to nine because it has to give one to nine to make ten. So nine plus seven is sixteen. I just know that pattern super fast. For eight plus five, I do make-a-ten fast, sort of just thinking five minus two is three, so thirteen. (Sam)
Sam: I know five and eight is thirteen, so why did you write a four in the tens column, Karen?	Because I had one more ten from the ones. See here in the drawing: nine ones and one one from the seven ones make ten ones. I wrote 10 here to remind me, and here in the problem I wrote the new one ten below where I can add it in after I find thirteen. You have to write your new one ten big enough to be sure you see it.
Sam: Oh yes, I see it now. I can see the new one ten when I write it, but I couldn't see yours.	OK, thanks. I'll write it bigger next time so everyone can see it.

The explainer stands to the side and points with a pointer to parts of the math drawing or to parts of the problem as they are mentioned. Pointing is a crucial part of the explanation.

FIGURE 5: *CCSS Addition (top row) and Multiplication (bottom row) Word Problem Situation and Math Expressions Diagrams for Each*

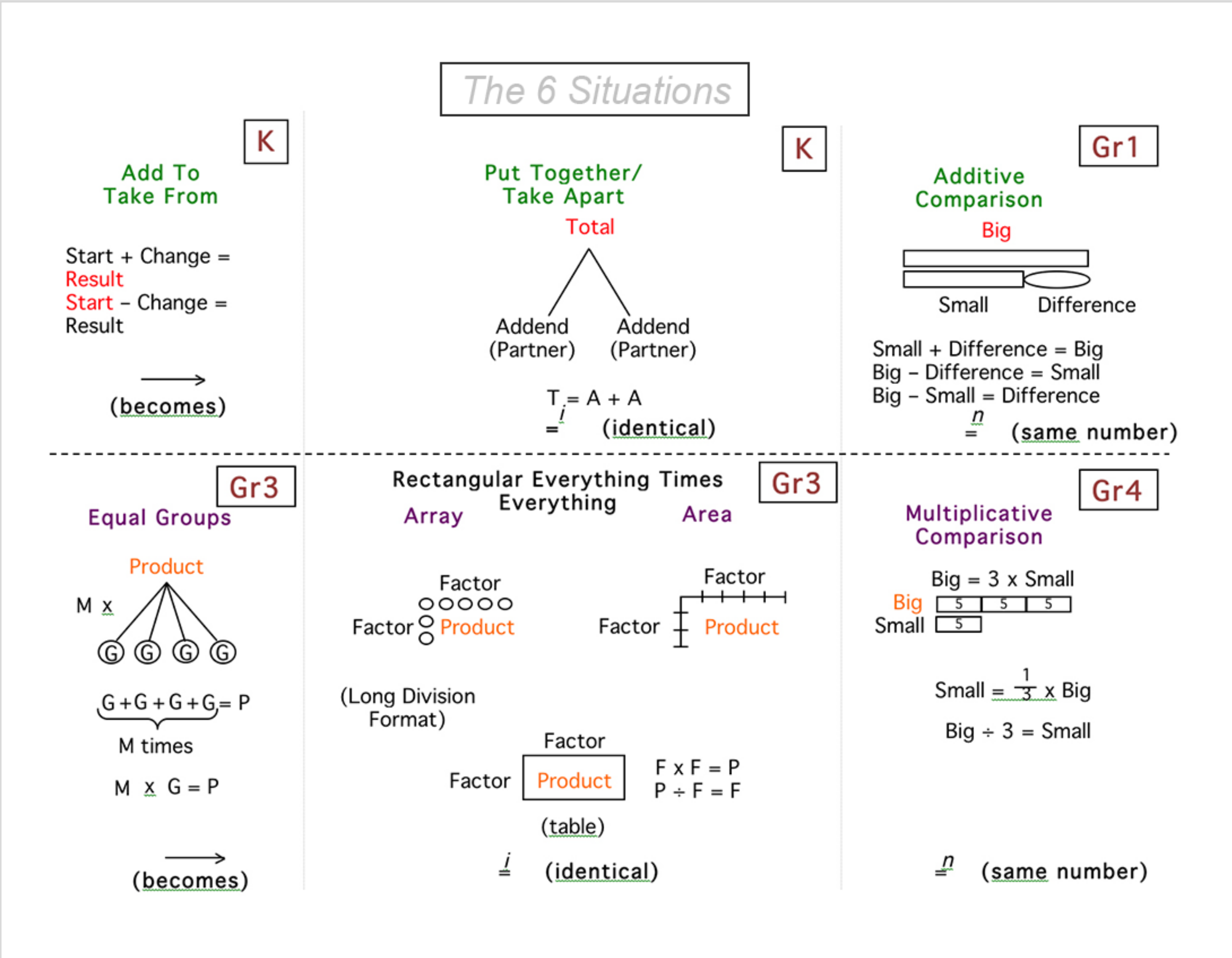


FIGURE 6: *Second Graders Modeling of a Situation Equation*

Levels of Labeled Math Drawings for an Add To: Start Unknown Problem

The key to solving story problems is understanding the situation. Students' equations often show the situation rather than the solution. Student drawings should be labeled to show which numbers or objects show which parts of the story situation.

1. Yolanda has a box of golf balls. Eddie took 7 of them. Now Yolanda has 5 left. How many golf balls did Yolanda have in the begining?