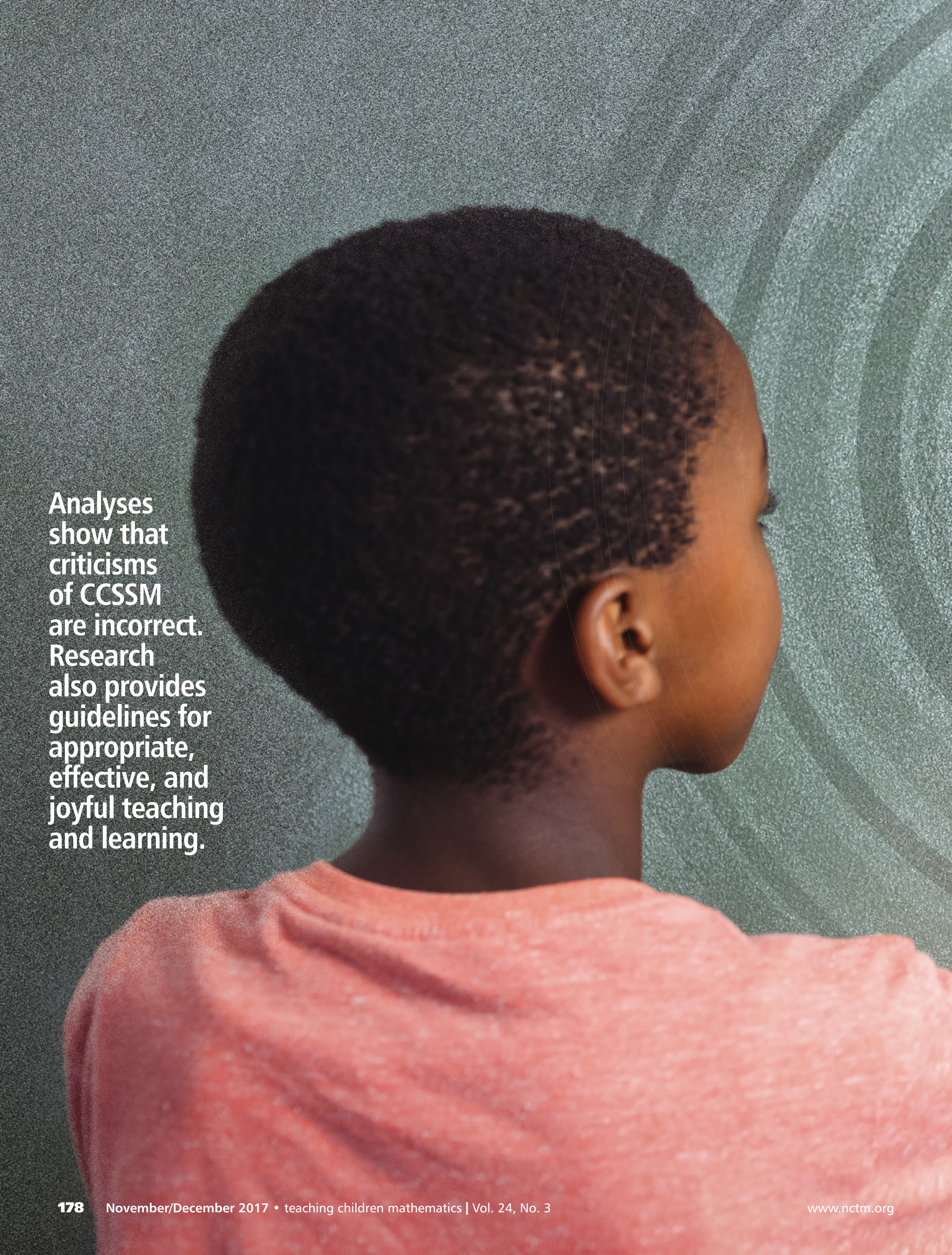


Analyses show that criticisms of CCSSM are incorrect. Research also provides guidelines for appropriate, effective, and joyful teaching and learning.

What Is Developmentally Appropriate Teaching?

WAVEBREAKMEDIA/THINKSTOCK

Douglas H. Clements,
Karen C. Fuson, and Julie Sarama

Alex is five years old. Her brother, Paul, is three.

Alex: When Paul is six, I'll be eight; when Paul is nine, I'll be eleven; when Paul is twelve, I'll be fourteen [*she continues until Paul is eighteen and she is twenty*].

Father: My word! How on earth did you figure all that out?

Alex: It's easy. You just go "three, *four*, five"; you go "six, *seven* [*clap*], eight"; you go "nine, *ten* [*clap*], eleven" (Davis 1984, p. 154).

Teachers are on the front line in any educational controversy. Increasingly, some bloggers, newspaper articles, and other media have criticized the Common Core State Standards for Mathematics (CCSSM) (CCSSI 2010) as being inappropriate for children in kindergarten and first grade. However, both research and expert practice reveal that children are capable of achieving these goals. In this article, we describe the results of our analysis of these criticisms in light of research, and we provide research-based guidelines for appropriate, effective, and joyful teaching and learning. We also include some comments about developmentally appropriate teaching for preschoolers so that they are not hindered from preparing adequately for kindergarten.

Criticisms of CCSSM

We found and analyzed four broad categories of criticisms of CCSSM (Clements, Fuson, and Sarama 2016):

1. No one who wrote the standards had any expertise in the education of very young children.
2. The standards are too early and therefore developmentally inappropriate for children in the early grades.
3. CCSSM dictates scripted curricula and didactic instruction rigidly applied to all children at the same pace.
4. CCSSM emphasizes academic skills and leaves no time for social-emotional development or play.

Our reviews of research and other documents found that none of these criticisms is accurate. Regarding criticism 1 (No one who wrote CCSSM had expertise in the education of very young children), although many people have claimed that early childhood educators were not involved in developing CCSSM (e.g., DEY 2014; for a list of other sources, see Clements, Fuson, and Sarama 2016), documentation that this is incorrect is readily available (Zimba 2015). Members of CCSSM feedback groups included public school early childhood and elementary teachers and directors of state programs, and their advice was used extensively. Also, we were involved in helping write CCSSM, and we have decades of experience working with teachers, children, and curricula from preschool through the primary grades.

The most frequent criticism of CCSSM for young children is that the standards are not "developmentally appropriate" (e.g., DEY 2014; Hess 2014; Strauss 2013; Walton 2014). However, these arguments are often based on misunderstandings of both CCSSM and children's development. Of special concern for us is that Kamii (2015) wrote an extensive criticism of standards in kindergarten and grades 1, 2, and 3. Kamii has helped many teachers learn about Piaget's groundbreaking research, carried out decades ago, about children's thinking. But the intervening decades have produced many research studies that have revised the original Piagetian results and have shown how much more children can do during these early years (Clements, Fuson, and Sarama 2016). Kamii did not include this research and so greatly underestimated what children can learn and what teachers can teach in developmentally appropriate ways. As an example, Kamii and others have claimed that expecting kindergartners to count to 100 is inappropriate (e.g., RealClearEducation 2014). However, even younger children learn principles, structure, and patterns in the number system as coded in their natural language, especially for number words above twenty (Baroody 1987; Fuson 1992a). Just as important, counting is interesting and important to children from ages two to five years (Gelman and Gallistel 1978), and if stimulated to do so, they love to count. They play with counting large numbers (Seo and Ginsburg 2004).

As another example, Kamii also criticizes the CCSSM objective of keeping correspondence when counting, claiming that children can do this only after they master the logical operations of hierarchical inclusion and seriation. However, full competence in these logical operations develops during the primary grades, and children can use one-to-one correspondences in counting years before, as the research has made clear (e.g., Clements and Sarama 2014; Fuson 1988). Therefore, Kamii's criticism and her suggestion that you cannot teach these competencies but only develop them indirectly, for example, by cleaning up spilled milk and playing Pick-Up Sticks, are incorrect. Children as young as four years of age can effectively be taught to count (Clements and Sarama 2014; Griffin, Case, and Capodilupo 1995).

The research foundation for CCSSM

CCSSM for kindergarten and grades 1 and 2 drew heavily on the research and recommendations of the National Research Council's (NRC) report "Mathematics Learning in Early Childhood" (2009). Subtitled "Learning Paths toward Excellence and Equity" to highlight the need for action to provide all children with high-quality learning opportunities, the report identified research-based foundational and achievable goals for prekindergarten, kindergarten, and grades 1 and 2; and the K–grade 2 goals were adapted in CCSSM. The major professional organizations concerned with the mathematical education of young children—the National Association for the Education of Young Children (NAEYC), the National Council of Teachers of Mathematics (NCTM), the National Council of Supervisors of Mathematics (NCSM), and the National Association of Early Childhood Specialists in State Departments of Education (NAECS-SDE)—all endorsed these CCSSM as being appropriate.

The foundational and achievable goals were drawn from the large international cognitive development and math education research on how children think about mathematical topics and what children can do at various ages (e.g., Clements and Sarama 2014; Fuson 1992a, 1992b; Sarama and Clements 2009). This research has shown that children can think more deeply than many of us knew before this research began. Alex's use of counting is but one example.

To illustrate CCSSM's appreciation for this depth of learning, we made a list of the main verbs in the kindergarten CCSSM, counted how many times each verb was used, classified the verbs into three categories, and counted the total for each category. Our results are listed below. Which do you think Alex was using?

- Three uses of mathematical skills: write numbers, name/identify, say
- Nineteen uses of mathematical actions: add and subtract, count, compare, compose/decompose, find the number, model, solve, sort
- Nineteen uses of mathematical thinking: analyze, classify, connect, describe, draw, represent, record by a drawing or equation, understand

The Standards for Mathematical Practice (SMP) in CCSSM further explicate the depth of learning in the standards. SMP "describe varieties of expertise that mathematics educators at all levels should seek to develop in their students" (CCSSI 2010, p. 6). There are eight of these practice standards, but eight is too many to keep in mind while teaching. These can be paired (SMP 1 and 6; SMP 2 and 3; SMP 4 and 5; and SMP 7 and 8—see **fig. 1**) and the pairs given names. They support a way to think about all eight mathematical practices in action simultaneously: *Teachers help children do meaning making* (SMP 1 and SMP 6) *about mathematical structure* (SMP 7 and SMP 8) *using math drawings* (including many visual supports, e.g., concrete objects, SMP 4 and SMP 5) *to support math explaining* (SMP 2 and SMP 3). We provide examples below.

FIGURE 1

Pairing the Common Core's Standards for Mathematical Practice (SMP) supports a way to think about all eight in action simultaneously: *Teachers help children do meaning making* (SMP 1 and SMP 6) *about mathematical structure* (SMP 7 and SMP 8) *using math drawings* (including many visual supports, e.g., concrete objects, SMP 4 and SMP 5) *to support math explaining* (SMP 2 and SMP 3). See CCSSI 2010, pages 6–8, for a full description of each practice.

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

The pairings above support a way to think about all eight SMP in action simultaneously: Teachers help children—

- do *meaning making* (SMP 1 and SMP 6)
- about *mathematical structure* (SMP 7 and SMP 8)
- *using math drawings* (including visual supports, e.g., concrete objects) (SMP 4 and SMP 5)
- to support *math explaining* (SMP 2 and SMP 3).

Standards for prekindergarten

CCSSM does not include prekindergarten, but teachers can help prekindergartners learn math concepts and skills that are the foundation for all later learning (NCTM 2010c includes a summary of the NRC report goals for pre-K). For preschoolers, the NRC report also recommended sustained focused math teaching (see **fig. 2**). The criticisms summarized above have

FIGURE 2

This summary of developmentally appropriate teaching-learning practices, which have appeared in various forms in books about effective teaching of mathematics in pre-K–grade 2, is jointly published by NCTM and the National Association for the Education of Young Children (NCTM 2010b, 2010c, 2010a, 2011).

Effective and developmentally appropriate teaching-learning practices

A. The teacher expects and supports **children's ability to make meaning and mathematize** the real world by—

- providing **settings that connect** mathematical language and symbols to quantities and to actions in the world;
- **leading children's attention** across these crucial aspects to help them make connections; and
- **supporting repeated experiences** that give children time and opportunity to build their ideas, develop understanding, and increase fluency.

B. The teacher creates a nurturing and helping **Math Talk Community**—

- within which to **elicit thinking** from children; and
- to help children **explain and help** each other explain and solve problems.

C. For each big math topic, the teacher leads the class through a **research-based learning path** based on children's thinking. This allows the teacher to differentiate instruction within whole-class, small-group, and center-based activities. This path provides the repetitive experiencing that young children need.

D. For later pre-K and kindergarten, children need to follow up activities with real, three-dimensional objects by working with math drawings and other written two-dimensional representations that **support children doing meaning-making about mathematical structure using math drawings to support math explaining**. Children of all ages also need to see and count groups of things in books, that is, they need to experience and understand three-dimensional things as pictures on a two-dimensional surface. Working with and on two-dimensional surfaces as well as with three-dimensional objects supports equity in math literacy because too many children have not had sufficient experiences with two-dimensional representations in their out-of-school environment.

also included pre-K, but we found no evidence that supported these criticisms.

Criticism 3 (CCSSM dictates scripted curricula and didactic instruction rigidly applied to all children at the same pace) is based on the incorrect notion that CCSSM presents not just *what* students should learn, but also *how* (e.g., http://www.allianceforchildhood.org/standards). However, standards do not dictate particular teaching methods (Tran et al. 2016). CCSSM's emphases on understanding and grounding in learning trajectories and progressions (http://commoncoretools.me/category/progressions/) are not particular teaching methods; they also are not consistent with inflexible curricula or teaching. As one example of the emphasis on understanding, CCSSM recommends that children "develop, discuss, and use" their own generalizable methods "using their understanding of place value and the properties of operations" (CCSSI 2010, p. 17). For example, a second grader might mentally solve 239 + 582 in this manner: "200 and 500 is 700. And 80 and 30 is another hundred and one more 10, so 800. Then 9, 10, 11. . . , and that other 10 is 21. So, 821." Notice how this child shows meaning making about mathematical structure. (We will describe another approach later in the article.) How can we promote such thinking?

A learning path view of teaching and learning

The NRC review (2009) found that teaching incidentally through play or only integrating math with other topics were insufficient. Sustained focused teaching and learning time for mathematics is essential. The report also summarized research about appropriate teaching-learning practices in early childhood that enable children to learn the foundational and achievable goals and to close the school entry knowledge gap.

These developmentally appropriate teaching-learning practices are consistent with *Principles to Actions: Ensuring Mathematical Success for All* (NCTM 2014). **Figure 2** summarizes these practices, which appeared in various related forms in books about effective teaching of mathematics in pre-K through grade 2, jointly published by NCTM and the National Association for the Education of Young Children (NAEYC) (NCTM 2010b, 2010c, 2010a, 2011). Those books also illustrate examples of the foundational and achievable goals in CCSSM and the NRC report—and of the crucial first step in *Principles to Actions*: "Establish mathematics goals to focus learning" (NCTM 2014, p. 10).

Parts A and B of **figure 2** require extensive and continual teaching actions by the teacher to model mathematical language connected to quantities, situations, and math objects and to help children make connections among these. This role is especially crucial in the ongoing nurturing and helping math talk community in which children share and explain their thinking and help one another explain and solve problems (see Hufferd-Ackles, Fuson, and Sherin [2015] for a summary). This math talk community—a continual teaching-learning environment in whole-class, small-group, and individual activities—supports children's need to hear mathematical language frequently and learn to use such mathematical language fluently to express their own thinking.

Children in prekindergarten and kindergarten need to work with real objects as they learn to count and carry out various mathematical actions and kinds of mathematical thinking. But, as summarized in part D of **figure 2**, they also need to work with two-dimensional pictures of things. Throughout grades 1 and 2, children can make simple math drawings, such as groups of circles, as a step in abstracting their mathematical thinking and supporting math explaining.

As emphasized in the first bullet in part A of **figure 2**, the crucial function of visual supports is to relate them to mathematical words and mathematical symbols so that the words and symbols gain meanings. Unfortunately, Bruner's (1966) modes of representations enactive (action-based)-iconic (image-based)-symbolic (language-based) is often still misinterpreted as a sequence in which these modes follow each other and are disconnected. But the point of the enactive and iconic is to provide meanings for the symbolic. They all can occur together, and they must be related. CCSSM mathematical practices capture these aspects and the need to build relationships among them, as in the single-sentence summary of the practices given above: Teachers help children do meaning-making about mathematical structure using math drawings to support math explaining. Children can use concrete objects instead of math drawings, but math drawings are useful in supporting explanations, and they leave a record of thinking. Teacher actions are crucial as they help children mathematize (see the mathematical structure) and elicit and support explaining.

The engaging and encouraging climate for learning envisioned in **figure 2** helps children develop confidence in their ability to understand and use mathematics. These positive experiences help children cultivate such dispositions as curiosity, imagination, flexibility, inventiveness, and persistence—which contribute to their future success in and out of school (e.g., Clements, Sarama, and DiBiase 2004). This climate is not created by simplifying what or how children learn but by giving them challenging learning opportunities and supporting their engagement with those opportunities and with one another.

Sustained, repeated experiences enable children to build conceptual relationships

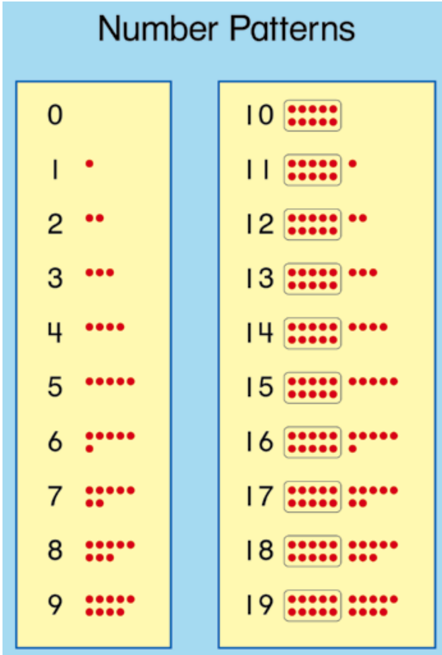
The third bullet in part A (see **fig. 2**) is especially important: supporting repeated experiences of making all the connections we have been describing. Children need many repetitions of a given experience to become fluent in making the visual and conceptual connections involved. For example, a visual display (see **fig. 3**) of how the single-digit numbers 0–9 relate to the teen numbers 10–19 could be used for the important CCSSM content standard K.NBT.1:

> Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

Before using this chart, children would have spent weeks seeing and making the numbers

FIGURE 3

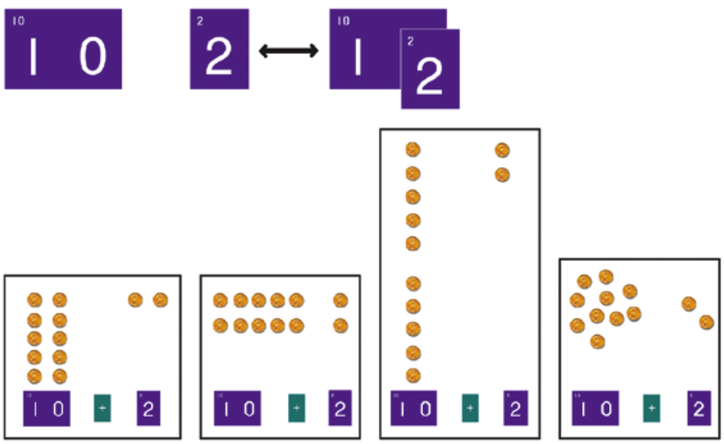
This is a visual display of how the single-digit numbers 0–9 relate to the teen numbers 10–19.



Math Expressions Kindergarten, "Number Pattern Poster, Unit 2, Lesson 10," p. 148. Reprinted with permission from Houghton Mifflin Harcourt.

FIGURE 4

The place-value conceptual web of relationships that students built in kindergarten and extended in grade 1 can culminate in grade 2 with using math drawings of place-value quantities—like these layered place-value teen cards and quantities as ten and two ones—to support adding and explaining their thinking.



Math Expressions Kindergarten, Unit 3, Lesson 5, p. 232. Reprinted with permission from Houghton Mifflin Harcourt.

six through ten using the five-based patterns (see **fig. 3**) and relating these patterns to their fingers: the one, two, three, four, and five repeat and are added to five to make six, seven, eight, nine, ten. Using these same patterns for the teen numbers to show one through nine enables children to see those teen numbers visually and to focus their attention on the new aspect of teen numbers: Each number has ten circles (seen immediately because of the two groups of five) and some ones.

Children can discuss patterns they see in this chart for many days so that all come to see the patterns and to articulate them. Children can enact these teen quantities by showing ten fingers to their left and then the ones quantities to their right, saying these quantities as they show them ten and one, ten and two, ten and three, . . . , ten and nine. They also can learn and practice saying the English words for the teen numerals eleven to nineteen as a child points to each numeral with its pattern of dots, so that the numerals, English words, and quantities gradually become related. Children can also say meaningful place value words for these numerals in order, with or without showing the ten fingers and several one's fingers: ten and one, ten and two, ten and three, . . . , ten and nine. Thus, gradually the teen numerals can take on the quantity meanings specified in the K.NBT.1 standard.

These quantity meanings can also be supported by the use of layered place-value cards (see **fig. 4**). These cards enable children to see the zero hiding under the digit in the ones place and thus to think of ten and some ones instead of the single digits 1 and 2 that they actually see in the number 12. Children can also count out various teen numbers with objects and in drawings and then group ten ones to see and say that teen number. Such activities can move from a disorganized group of ten to ten as two fives (see **fig. 3**) to a vertical column of two fives that is moving toward the grade 1 concept of ten as one group of ten (see **fig. 4**).

Early childhood educators suggested to the writers of CCSSM that teen numbers in kindergarten be conceptualized as ten ones and that the more difficult concept of one ten be saved for grade 1. The writers did so (see K.NBT.1 and 1.NBT.2). The place-value conceptual web of relationships built in kindergarten and extended in grade 1 can culminate in grade 2 with children using math drawings of place-value quantities, hundred boxes, tens sticks, and ones circles to support adding and explaining their thinking (see **fig. 5** on **p. 186**). First graders can make similar drawings and explanations for a range of methods for adding with regrouping within 100 (1.NBT.4).

We see how the patterns and relationships require the assistance of the teacher to orchestrate children's seeing and explaining patterns as well as enacting and building or drawing quantities related to number words and written symbols. Other topics, such as problem solving and geometry, also require extended teaching-learning sequences (see the NCTM/NAEYC books for summaries). Much of this orchestration needs to be done in a whole-class setting so that all children can interact with these ideas with the support of the teacher. Small groups, partners, and individual work also have roles in the classroom, but the sustained whole-group activities are key in the early grades. Research and experience with CCSSM indicate that an hour a day is crucial to help all children be successful.

Regarding criticism 4 (CCSSM emphasizes academic skills and leaves no time for social-emotional development or play), "play versus academic teaching" is a false dichotomy that harms our children (Clements, Fuson, and Sarama 2016). This false dichotomy is related to the myth in criticism 3: that CCSSM teaching must be didactic. But we have seen that teaching high-quality math standards can support social competencies as children think for themselves, explain their ideas, and play with mathematical concepts and language. Such teaching can support positive social interactions, build a range of math competencies, and build language and self-regulation abilities. Children in prekindergarten, kindergarten, and first grade deserve research-based teaching and learning in which they can become deeply engaged and so become confident and competent.

Final words

In summary, developmentally appropriate practice (DAP) does not mean age-based limitations (cf. Kamii 2015, p. 12). What is developmentally inappropriate are the many present-day kindergarten curricula that "teach" most children what they already know (e.g., Carpenter and Moser 1984; Engel, Claessens, and Finch 2013; Van den Heuvel-Panhuizen 1996) or the many preschools that teach very little math (Ginsburg, Klein, and Starkey 1998; Graham, Nash, and Paul 1997; Tudge and Doucet 2004), especially when successful research-based approaches are available that help children learn so much more (Clements and Sarama 2011).

How we achieve them was our final focus. Not all children are provided with the teaching, materials, and tools needed to fulfill their potential for learning math—a critical equity issue in the United States (Morgan et al. 2014; NRC 2009). We must provide these learning and teaching resources everywhere we can. Thus, what DAP does mean is captured in the title of the NRC (2009) report: "Mathematics in Early Childhood: Learning Paths toward Excellence and Equity." Teachers of young children can be confident that teaching mathematics along learning trajectories, with the practices in **figure 2**, fully realize NAEYC and NCTM's definition of truly developmentally appropriate learning and teaching: challenging but achievable.

Common Core Connections

ALL K–GRADE 2 CONTENT STANDARDS

ALL EIGHT SMP

REFERENCES

Baroody, Arthur J. 1987. *Children's Mathematical Thinking*. New York: Teachers College.

Bruner, Jerome. 1966. *Toward a Theory of Instruction*. Cambridge, MA: Harvard University Press.

Carpenter, Thomas P., and James M. Moser. 1984. "The Acquisition of Addition and Subtraction Concepts in Grades One through Three." *Journal for Research in Mathematics Education* 15, no. 3 (May): 179–202.

Common Core State Standards Initiative (CCSSI). 2010. Common Core State Standards for Mathematics (CCSSM). Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers. http://www.corestandards.org/wp-content/uploads/Math_Standards.pdf

Clements, Douglas H., Karen C. Fuson, and Julie Sarama. "The Common Core in Early Math and Developmental Appropriateness." Under review.

FIGURE 5

The explainer stands to the side and uses a pointer to note parts of the math drawing or problem as they are mentioned. Pointing is a crucial part of the explanation.

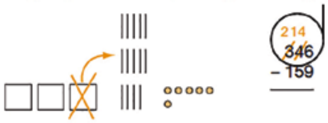
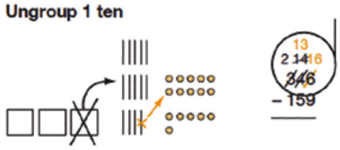
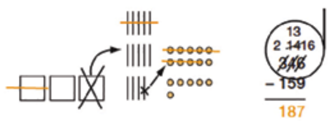
Math Drawing and Problem	Explanation Using Place-Value Language About Hundreds, Tens, and Ones
a. **Write problem and make drawing** 	I drew three hundreds, four tens, and six ones to show three hundred forty six. I wrote one hundred,five tens, nine ones below three hundred forty six in my problem, but I did not draw it because it is already part of three hundred forty six. To subtract, we separate the total into two numbers, the number we are taking away and the number that is left. Here I drew my magnifying glass around the total to remind me to check if I need to ungroup to get more to subtract.
b. **Ungroup 1 hundred (solving left to right)** 	I checked to see if I need to ungroup left to right. I can do it right to left too. So here in the hundreds I can take one hundred from three hundreds, so that column is OK. In the tens column, I cannot take five tens from four tens because five is more than four. So I need to get more tens to go with my four tens. I open up one hundred to make it be ten tens. Here I wrote my ten tens in two rows of five so you can see them clearly. And in my problem I showed that ungrouping by crossing out the three hundreds and writing the two hundreds I have left. And the four tens become fourteen tens here. So I'll be able to subtract the tens.
c. **Ungroup 1 ten** 	Now I check to see if I can subtract the ones. Nope. Nine is more than six, so I need to get more ones also. I open up one ten here to show that it has ten ones hiding in it. I write them in two rows of five so I know I made exactly ten and you can see them. In my problem I ungrouped by taking one ten from the fourteen tens and writing thirteen above in the tens column. And the ten ones make sixteen ones with the six, so I write sixteen at the top of the ones column.

Fig. 2.22. Student explanation and class discussion for generalizable and accessible ungroup-first-where-needed method for subtracting 3-digit numbers Part I

Math Drawing and Problem	Explanation Using Place-Value Language About Hundreds, Tens, and Ones
d. **Subtract H, T, O L to Rt or Rt to L** 	Now I can subtract in every column. I can go in either direction. I'll go left to right again. I take away one hundred in my drawing, and one hundred is left. My problem agrees: I take away one hundred from the two hundreds and write the one hundred that is left. I'll subtract five tens from thirteen tens and get eight tens. I just know that. But here in my drawing I'll take the five tens from the ten tens, and I can do make-a-ten if I don't know thirteen minus five. See, five more left in the ten and the three in thirteen make eight. For the ones I can use Karen's pattern she just explained, that the teen total is one less than the ones added on to a nine. So sixteen minus nine is seven. See here in the drawing, you can see the one extra with the nine that gets added to the six ones to make seven ones. Are there any questions? Yes, Sybilla?
Student Question	**Explainer Answer**
Sybilla: Doug, why didn't you subtract six ones from nine ones to get three ones in the answer?	Because we have to subtract the addend from the total. Six is part of the total, so we have to subtract from it. But we can't, so that's why I had to get more ones here. Good question, even though I know you know this. Hank?
Hank: What if you checked your hundreds and the bottom number was bigger? How could you subtract?	I couldn't. The total has to be bigger than the addend I subtract because that addend is just part of the total. But sometimes I write the numbers backwards, so I check the problem again if I can't subtract the hundreds. Efrain?
Efrain: How would your problem be different if you had ungrouped right to left?	Only the tens place would look different. Remember how we did it both ways and talked about this yesterday? And look at Yeping's problem. He ungrouped right to left. The tens place looks different because you ungroup one ten to make ten ones before you get ten tens. So you write three and then thirteen. But I end up with thirteen, so the ungrouping gets the same number in each column ready to subtract.

The explainer stands to the side and points with a pointer to parts of the math drawing or to parts of the problem as they are mentioned. Pointing is a crucial part of the explanation.

Fig. 2.22. Student explanation and class discussion for generalizable and accessible ungroup-first-where-needed method for subtracting 3-digit numbers Part II—*Continued*

Focus in Grade 2: Teaching with Curriculum Focal Points, figure 2.22, pp. 88 and 89. Reprinted with permission from the National Council of Teachers of Mathematics.

Clements, Douglas H., and Julie Sarama. 2011. "Early Childhood Mathematics Intervention." *Science* 333 (6045): 968–70. doi:10.1126/science.1204537

———. 2014. *Learning and Teaching Early Math: The Learning Trajectories Approach*. 2nd ed. New York: Routledge.

Clements, Douglas H., Julie Sarama, and Ann-Marie DiBiase. 2004. *Engaging Young Children in Mathematics: Standards for Early Childhood Mathematics Education*. Mahwah, NJ: Lawrence Erlbaum Associates.

Davis, Robert B. 1984. *Learning Mathematics: The Cognitive Science Approach to Mathematics Education*. Norwood, NJ: Ablex.

Defending the Early Years (DEY). "Six Reasons to Reject the Common Core State Standards for K–Grade 3," May 1, 2014. http://www.deyproject.org/dey-archives/september-19th-20168628820

Engel, Mimi, Amy Claessens, and Maida A. Finch. 2013. "Teaching Students What They Already Know? The (Mis)Alignment between Mathematics Instructional Content and Student Knowledge in Kindergarten." *Educational Evaluation and Policy Analysis* 35 (2):157–78. doi:10.3102/0162373712461850

Fuson, Karen C. 1988. *Children's Counting and Concepts of Number*. New York: Springer-Verlag.

———. 1992a. "Research on Learning and Teaching Addition and Subtraction of Whole Numbers." In *Analysis of Arithmetic for Mathematics Teaching*, edited by *Gaea Leinhardt, Ralph T. Putnam, and Rosemary A. Hattrup*, pp. 53–187. Mahwah, NJ: Lawrence Erlbaum Associates.

———. 1992b. "Research on Whole Number Addition and Subtraction." In *Handbook of Research on Mathematics Teaching and Learning*, edited by Douglas A. Grouws, pp. 243–75. New York: Macmillan.

Gelman, Rochel, and Charles Ransom Gallistel. 1978. *The Child's Understanding of Number*. Cambridge, MA: Harvard University Press.

Ginsburg, Herbert P., Alice Klein, and Prentice Starkey. 1998. "The Development of Children's Mathematical Thinking: Connecting Research with Practice." In *Handbook of Child Psychology, Volume 4: Child Psychology in Practice*, edited by William Damon, Irving E. Sigel, and K. Ann Renninger, pp. 401–76. New York: John Wiley and Sons.

Graham, Theresa A., Cindy Nash, and Kim Paul. 1997. "Young Children's Exposure to Mathematics: The Child Care Context." *Early Childhood Education Journal* 25 (1): 31–38.

Griffin, Sharon, Robbie Case, and Allesandra Capodilupo. 1995. "Teaching for Understanding: The Importance of the Central Conceptual Structures in the Elementary Mathematics Curriculum." In *Teaching for Transfer: Fostering Generalization in Learning*, edited by Anne McKeough, Judy Lupart, and Anthony Marini, pp. 121–51. Mahwah, NJ: Lawrence Erlbaum Associates.

Hess, Frederick M. 2014. "How the Common Core Went Wrong." *National Affairs* 21 (Fall).

Hufferd-Ackles, Kimberly, Karen C. Fuson, and Miriam Gamoran Sherin. 2015. "Describing Levels and Components of a Math-Talk Learning Community." *Journal for Research in Mathematics Education* 35:81–116.

Kamii, Constance. "Selected Standards from the Common Core State Standards for Mathematics, Grades K–3: My Reasons for Not Supporting Them." *Defending the Early Years*, May 13, 2015. http://www.deyproject.org/uploads/1/5/5/7/15571834/kamiideyccssmathpaper.pdf

Math Expressions Kindergarten. 2013. Boston: Houghton Mifflin Harcourt.

Let's chat!

On the second Wednesday of each month, *TCM* hosts a lively discussion with authors and *TCM* readers about a topic important in our field.

You are invited to participate in the fun. On **Wednesday, November 8, 2017**, at **9:00 p.m. EST**, we will discuss "What Is Developmentally Appropriate Teaching?" by Douglas H. Clements, Karen C. Fuson, and Julie Sarama.

Follow along using #tcmchat or follow us on twitter@tcm_at_nctm and watch for a link to the recap.

Morgan, Paul L., George Farkas, Marianne M. Hillemeier, and Steve Maczuga. 2016. "Who Is at Risk for Persistent Mathematics Difficulties in the United States?" *Journal of Learning Disabilities* 49 no. 3 (May–June): 305–19. doi:10.1177/0022219414553849

National Council of Teachers of Mathematics (NCTM). 2010a. *Focus in Grade 1: Teaching with the Curriculum Focal Points*. Reston, VA: NCTM.

———. 2010b. *Focus in Kindergarten: Teaching with the Curriculum Focal Points*. Reston, VA: NCTM.

———. 2010c. *Focus in Prekindergarten: Teaching with the Curriculum Focal Points*. Reston, VA: NCTM.

———. 2011. *Focus in Grade 2: Teaching with the Curriculum Focal Points*. Reston, VA: NCTM.

———. 2014. *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: NCTM.

National Research Council (NRC). 2009. *Mathematics in Early Childhood: Paths toward Excellence and Equity*, edited by Christopher T. Cross, Taniesha A. Woods, and Heidi Schweingruber. Washington, DC: National Academies Press.

RealClearEducation. 2014. "Common Core 'Goes Way, Way Too Far': 5 Questions with N.Y. Principal Carol Burris." *RealClearEducation*. Accessed April 18, 2016.

Sarama, Julie, and Douglas H. Clements. 2009. *Early Childhood Mathematics Education Research: Learning Trajectories for Young Children*. New York: Routledge.

Seo, Kyoung-Hye, and Herbert P. Ginsburg. 2004. "What Is Developmentally Appropriate in Early Childhood Mathematics Education?" In *Engaging Young Children in Mathematics: Standards for Early Childhood Mathematics Education*, edited by Douglas H. Clements, Julie Sarama, and Ann-Marie DiBiase, pp. 91–104. Mahwah, NJ: Lawrence Erlbaum Associates.

Strauss, Valerie. "Answer Sheet: A Tough Critique of Common Core on Early Childhood Education," *The Washington Post*, January 29, 2013. https://www.washingtonpost.com/news/answer-sheet/wp/2013/01/29/a-tough-critique-of-common-core-on-early-childhood-education/

Tran, Dung, Barbara J. Reys, Dawn Teuscher, Shannon Dingman, and Lisa Kasmer. 2016. "Analysis of Curriculum Standards: An Important Research Area." *Journal for Research in Mathematics Education* 47, no. 2 (March): 118–33.

Tudge, Jonathan R. H., and Fabienne Doucet. 2004. "Early Mathematical Experiences: Observing Young Black and White Children's Everyday Activities." *Early Childhood Research Quarterly* 19 (1): 21–39.

Van den Heuvel-Panhuizen, Marja. 1996. *Assessment and Realistic Mathematics Education*. Utrecht, The Netherlands: Freudenthal Institute, Utrecht University.

Walton, Alice G. "The Science of the Common Core: Experts Weigh in on Its Developmental Appropriateness." *Forbes*, October 23, 2014. http://www.forbes.com/sites/alicegwalton/2014/10/23/the-science-of-the-common-core-experts-weigh-in-on-its-developmental-appropriateness/

Zimba, Jason. 2015. "The Developmental Appropriateness of the Common Core State Standards for Mathematics." *Student Achievement Partners*. https://achievethecore.org/file/2644

This article was supported in part by the Institute of Education Sciences, U.S. Department of Education through Grants No. R305K05157 and No. R305A110188, the National Science Foundation through Grant No. DRL-1313695, the Gates Foundation, and the Heising-Simons Foundation. The opinions expressed are those of the authors and do not represent views of these agencies.

The first two authors advised the writers of CCSSM. **Douglas H. Clements**, **Douglas.Clements@du.edu**, is Kennedy Endowed Chair in Early Childhood Learning; co-executive director of the Marsico Institute; and a professor at the University of Denver in Colorado. **Karen C. Fuson**, **karenfuson@mac.com**, is a professor emerita at Northwestern University in Evanston, Illinois, and now lives in Fallbrook, California. **Julie Sarama**, **Julie.Sarama@du.edu**, is Kennedy Endowed Chair in Innovative Learning Technologies, co-executive director of the Marsico Institute for Early Learning and Literacy, and a professor at the University of Denver. All three authors are interested in early and elementary mathematics education as well as research and development in curriculum development, teaching, and professional development.

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

PREMIER MATH EDUCATION EVENT

NCTM ANNUAL MEETING & EXPOSITION 2018

April 25–28 | Washington, DC

Empowering the Mathematics Community

REGISTER NOW

It's never too early to plan ahead for the leading math education event of the year. Network with thousands of your peers and fellow math education professionals to exchange ideas, engage with innovation in the field, and discover new learning practices that will drive student success.

The latest teaching trends and topics will include:

- **Tools and Technology:** Enhancing Instruction and Promoting Innovation
- **Access, Equity, and Empowerment:** Transformative Practices and Professional Accountability
- **Purposeful Curriculum:** Cultivating Coherence and Connections
- **Teaching and Learning:** Building a Community of Empowered Learners
- **Assessment:** Involving and Empowering Students
- **Professionalism:** Empowering Teachers through Community
- **Mathematical Modeling:** Interpreting the World through Mathematics
- **Emerging Issues and Hot Topics**

The NCTM Annual Meeting & Exposition is ideal for:

- PRE-K–12 TEACHERS
- MATH TEACHER EDUCATORS
- NEW AND PROSPECTIVE TEACHERS
- MATH COACHES AND SPECIALISTS
- MATH RESEARCHERS
- SCHOOL AND DISTRICT ADMINISTRATORS

Learn more at **nctm.org/annual**

and follow us on f in You Tube #NCTMannual