Children Living in Poverty Can Solve CCSS OA Word Problems

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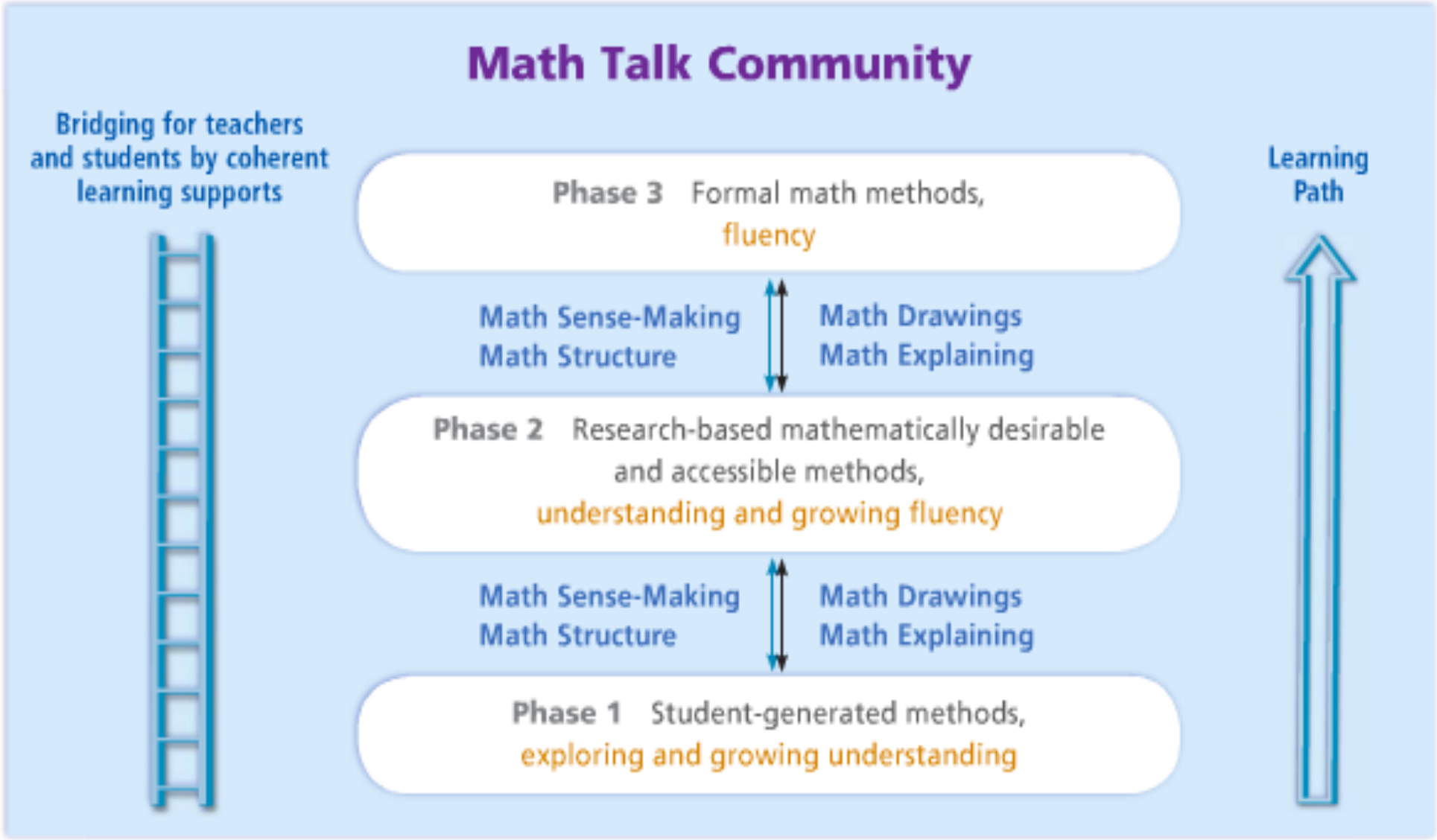
For more details about the CCSS-M and visual supports, please see the series of visual with audio Teaching Progressions I have made for various math domains. These can be found at karenfusonmath.com


Math Talk Community
Bridging for teachers and students by coherent learning supports
Phase 3 Formal math methods, fluency
Math Sense-Making
Math Structure
Math Drawings
Math Explaining
Phase 2 Research-based mathematically desirable and accessible methods, understanding and growing fluency
Math Sense-Making
Math Structure
Math Drawings
Math Explaining
Phase 1 Student-generated methods, exploring and growing understanding
Learning Path

Common Core Mathematical Practices Used in a Math Talk Community

Math Sense-Making: Make sense and use appropriate precision 1 Make sense of problems and persevere in solving them. 6 Attend to precision.	**Math Drawings: Model and use tools** 4 Model with mathematics. 5 Use appropriate tools strategically.
Math Structure: See structure and generalize 7 Look for and make use of structure. 8 Look for and express regularity in repeated reasoning.	**Math Explaining: Reason, explain, and question** 2 Reason abstractly and quantitatively. 3 Construct viable arguments and critique the reasoning of others.

Figure 2

The Math Practices in action

A teacher asks every day:

Did I do math sense-making about math structure using math drawings to support math explaining?

Can I do some part of this better tomorrow?

OA: Operations and Algebraic Thinking

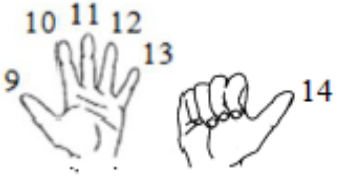
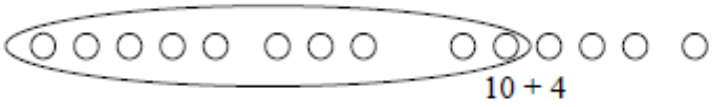
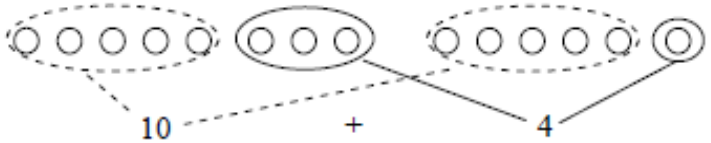
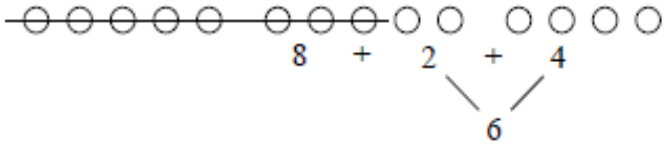
Learning paths within and across grades for

- situations (problem types) that give meanings for operations
- single-digit computation (+- and x÷)

Students represent using drawings/diagrams and/or equations, then solve.

Students understand and apply properties of operations and the relationship between addition/subtraction and multiplication/division).

Levels of Children's Addition and Subtraction Methods

Levels	8 + 6 = 14	14 – 8 = 6
Level 1: Count all	Count All a b 1 2 3 4 5 6 7 8 1 2 3 4 5 6 1 2 3 4 5 6 7 8 9 10 11 12 13 14 c	Take Away a 1 2 3 4 5 6 7 8 9 10 11 12 13 14 1 2 3 4 5 6 7 8 1 2 3 4 5 6 b c
Level 2: Count on	Count On 8 8 9 10 11 12 13 14	To solve 14 – 8 I count on 8 + ? = 14 I took away 8  8 to 14 is 6 so 14 – 8 = 6
Level 3: Recompose Make a ten (general): one addend breaks apart to make 10 with the other addend Make a ten (from 5's within each addend)	Recompose: Make a Ten   	14 – 8: I make a ten for 8 + ? = 14  8 + 6 = 14
Doubles ± *n*	6 + 7 = 6 + 6 + 1 =12 + 1 = 13	6 + ? = 13 = 6 + 6 + 1 = 12 + 1= 13

Note: Many children attempt to count down for subtraction, but counting down is difficult and error-prone. Children are much more successful with counting on; it makes subtraction as easy as addition.

Concepts for Each Level

Level 1: Each new addend and the total are separate.

Level 2: Both addends are embedded within the total.

Level 3: Addends are recomposed to make new addends (e.g. 8+6 becomes 10 + 4)

Make-a-ten prerequisites

a. Partner of the larger addend to 10 (K.OA.4)
b. All partners of the smaller addend to find how much over ten (K.OA.3)
c. 10 + n for n = 1 to 9 (K.NBT.1)

	Result Unknown	Change Unknown	Start Unknown
Add To	*A* bunnies sat on the grass. *B* more bunnies hopped there. How many bunnies are on the grass now? A + B = ☐ **K**	*A* bunnies were sitting on the grass. Some more bunnies hopped there. Then there were *C* bunnies. How many bunnies hopped over to the first two? A + ☐ = C **1**	Some bunnies were sitting on the grass. *B* more bunnies hopped there. Then there were *C* bunnies. How many bunnies were on the grass before? ☐ + B = C **2**
Take From	*C* apples were on the table. I ate *B* apples. How many apples are on the table now? C – B = ☐ **K**	*C* apples were on the table. I ate some apples. Then there were *A* apples. How many apples did I eat? C – ☐ = A **1**	Some apples were on the table. I ate *B* apples. Then there were *A* apples. How many apples were on the table before? ☐ – B = A **2**

	Total Unknown	Both Addends Unknown[1]	Addend Unknown[2]
Put Together /Take Apart	*A* red apples and *B* green apples are on the table. How many apples are on the table? A + B = ☐ **K**	Grandma has *C* flowers. How many can she put in her red vase and how many in her blue vase? C = ☐ + ☐ **K**	*C* apples are on the table. *A* are red and the rest are green. How many apples are green? A + ☐ = C C – A = ☐ **1**

	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare	*"How many more?" version.* Lucy has *A* apples. Julie has *C* apples. How many more apples does Julie have than Lucy? **1** *"How many fewer?" version.* Lucy has *A* apples. Julie has *C* apples. How many fewer apples does Lucy have than Julie? A + ☐ = C C – A = ☐ **1**	*"More" version suggests operation.* Julie has *B* more apples than Lucy. Lucy has *A* apples. How many apples does Julie have? **1** *"Fewer" version suggests wrong operation.* Lucy has *B* fewer apples than Julie. Lucy has *A* apples. How many apples does Julie have? A + B = ☐ **2**	*"Fewer" version suggests operation.* Lucy has *B* fewer apples than Julie. Julie has *C* apples. How many apples does Lucy have? **1** *"More" version suggests wrong operation.* Julie has *B* more apples than Lucy. Julie has *C* apples. How many apples does Lucy have? C – B = ☐ ☐ + B = C **2**

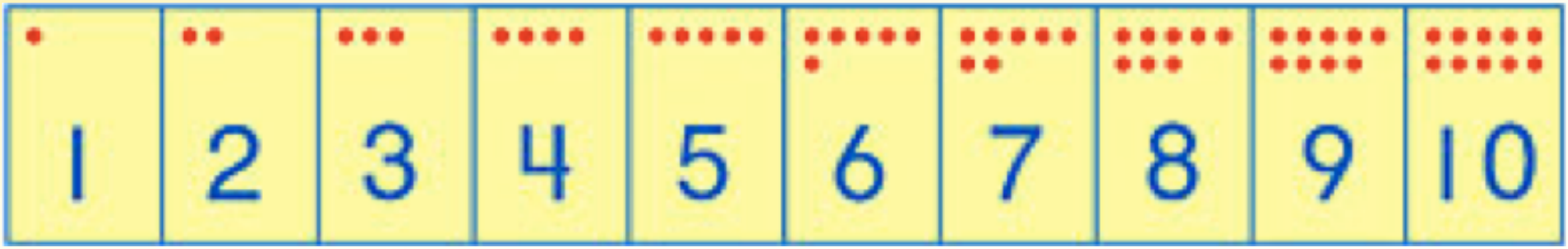
Represent the Situation
OA: Operations and Algebraic Thinking

Grade 1 and Grade 2 subtypes involve algebraic thinking:

Represent the situation with a drawing, diagram, and/or an equation.

Then decide how to solve for the answer.

K and 1 Seeing 1 to 10



This large Number Parade was on the wall and used in many activities.

See the 5-groups that make 6 through 10.

K U1 Working with numbers 1 to 5

Unit 1: Activities to 5 and then to 10.

A) Put number tiles in order at top and 5 red and 5 blue tiles at bottom.

B) Pull down the number tile for the number said.

C) Show that number of tiles.

D) Have a Math Talk Discussion:

a. relate the visual quantity to fingers, sounds, and body movements

b. practice visual imagery (Close your eyes. Visualize.)

c. describe different arrangements by color, dot/no dot, spatial relationships (e.g., 3 = 2 + 1)

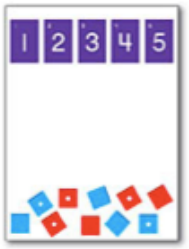
d. change your arrangement and discuss why you still have 3

e. copy the arrangement of another person

f. see partners of numbers already described in c and create new partners

g. graph on a graph map (2 rows/columns of 10 empty squares).

A)



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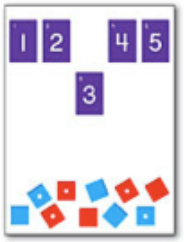
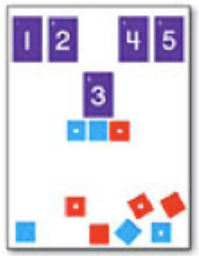


C)



K U2 Working with 5-groups

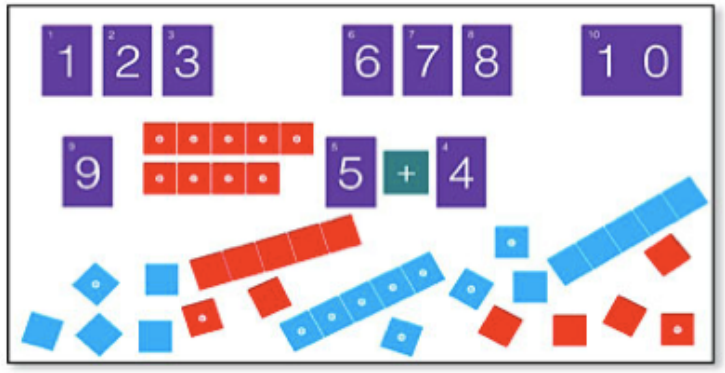
Unit 2: Use 5-groups to show quantities, addition expressions, and total for numbers 6 to 10

Use Unit 1 Steps A, B, C with a group of 5 and some units:

one unit of 5 red or blue squares, each with a dot on one side or

one unit of 5 pennies drawn in squares on a strip.

Children put tiles for the total to the left and for an addition expression for the partners (addends) to the right.

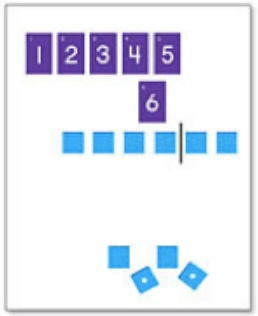


K U3 Working with partners (addends)

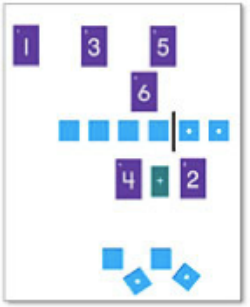
Unit 3: Partners of 2, 3, 4, 5, and 6 with tiles, break-apart stick, total, and addition expression

A) Make a number with a numeral tile and that many things.
B) Elicit partners of that number.
C) Use a break-apart stick to show the partners.
D) Use number tiles and the + tile to show an addition expression for the partners and say the partners: Six is four plus two. Show with fingers. Teacher writes equation 6 = 4 + 2.
E) Switch the partners with objects, stick, and tiles. Teacher writes 6 = 2 + 4 beside 6 = 4 + 2.
F) Repeat for different partners of the number.
G) Repeat all steps with a different number.

A-C)

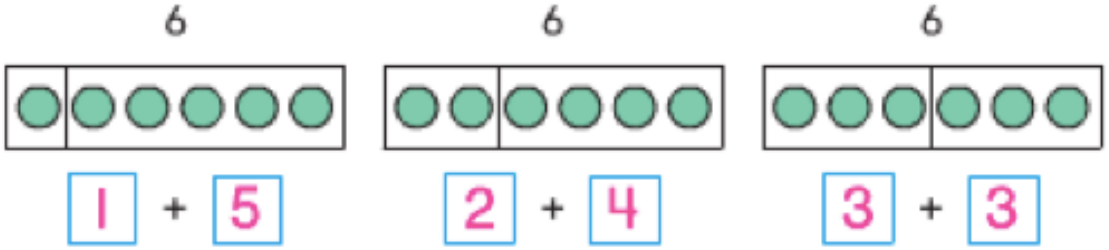


D)

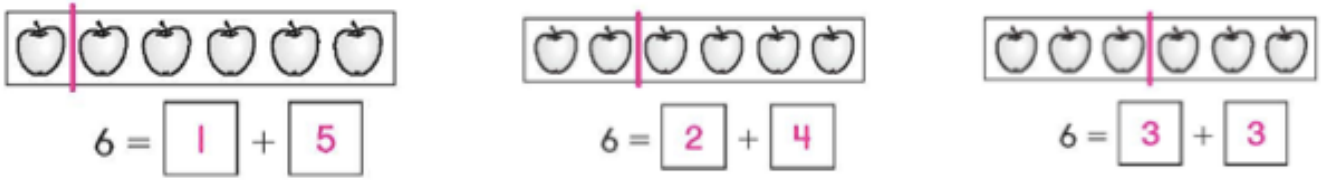


K Partner (addend) unit test results

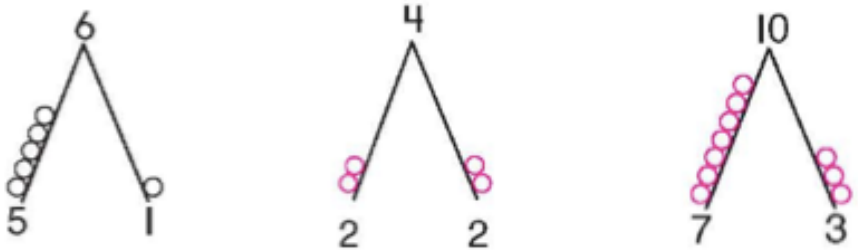
3 90 1. Write the Partners



4 92 2. Draw a line to show the partners. Write the partners.

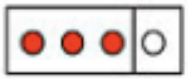


4 92 3. Draw Tiny Tumblers on the Math Mountain

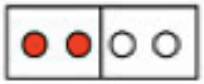


K Partner (addend) unit test results

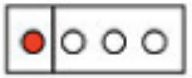
4 85 4. Write the partner equation.



4 = 3 + 1



4 = 2 + 2

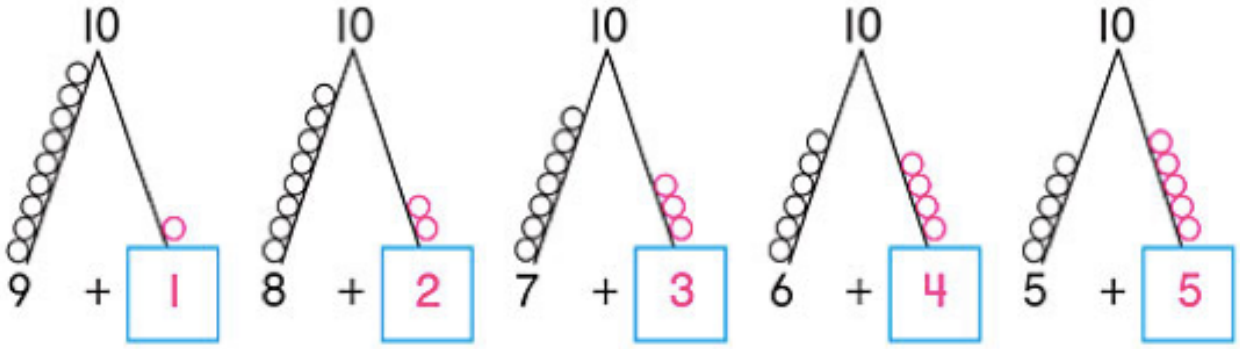


4 = 1 + 3

5 88 5. Shade to show all the 5-partners in order. Write the 5-partners.

1 + 4 2 + 3 3 + 2 4 + 1

5 83 6. Draw Tiny Tumblers on the Math Mountain and write the partner.



K Word Problem Solving Progression

↑

B) Mathematize situations from student lives and from pictures of scenes to describe word problem situations and ask questions.

↑

A) See and draw groups of 1 to 5 things in a real-world scene, then 6 to 10 things.

Mathematizing Real-World Situations

1 to 5

↑

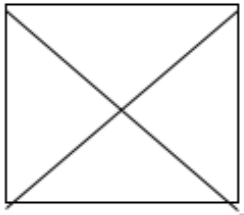
B) Teacher and children act out each situation with objects and fingers.

↑

A) Work on numerical triad relationships for 1 to 5 and then 6 to 10 with 5-groups.

o o o o o six 6
o

Situation and Solution Repesentations



Mathematical Language

Five has three and two inside.
Four plus three equals seven.
Seven minus four equals three.
Put together, add, and, in all, total, together, altogether, in the beginning, then, at the end, take away, left, rest

Written Mathematical Marks

Expressions	Equations	
3 + 2	5 = 3 + 2	
4 + 3	4 + 3 = 7	4 + 3 = ☐
7 – 4	7 – 4 = 3	7 – 4 = ☐

Teacher writes → child makes with number and +/- tiles → child writes

K Word Problem Solving Progression

E) Children tell and retell word problems for any situation; continue to focus on language.

↑

D) Continue approaches below using new familiar settings. Retell and work on language and on the question.

↑

C) Elicit word problems from a familiar setting and focus on/relate/extend the language: Children retell in different words and practice the question.

↑

B) Mathematize situations from student lives and from pictures of scenes to describe word problem situations and ask questions.

↑

A) See and draw groups of 1 to 5 things in a real-world scene, then 6 to 10 things.

Totals of

6 to 10

6 to 10

1 to 5

D) Children make math drawings and also solve by fingers. Children write expressions or equations. Teacher writes equations and elicits and summarizes all connections.

↑

C) Each child solves with fingers or objects and shows the expression with number tiles. Children share and discuss solutions. Teacher makes math drawings and writes equations and relates all aspects.

↑

B) Teacher and children act out each situation with objects and fingers.

↑

A) Work on numerical triad relationships for 1 to 5 and then 6 to 10 with 5-groups.

o o o o o six 6

o

Half-Day Kindergarten

Task	Stigler, Lee, Stephenson			*Math Expressions* Half-Day
	Japanese Grade 1 $n = 120$	Chinese Grade 1 $n = 120$	U.S. Grade 1 $n = 240$	Kinder-garten $n = 68$
Addition				
5 + 4	99	96	77***	100
3 + 2 word problem: Joey had 3 marbles and then found 2 more. How many marbles does Joey have now?	98	97	89***	97
Subtraction				
9 - 1	80	74	52***	81
6 – 2 word problem: Jan's father gave her 6 cookies. She ate 2 of them. How many did she have left?	93	81	73*	90

Grade 1 Partner Switches

Show the 8-partners and switch the partners.

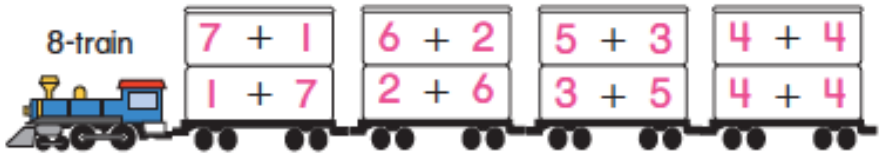
1.  7 + 1 and 1 + 7

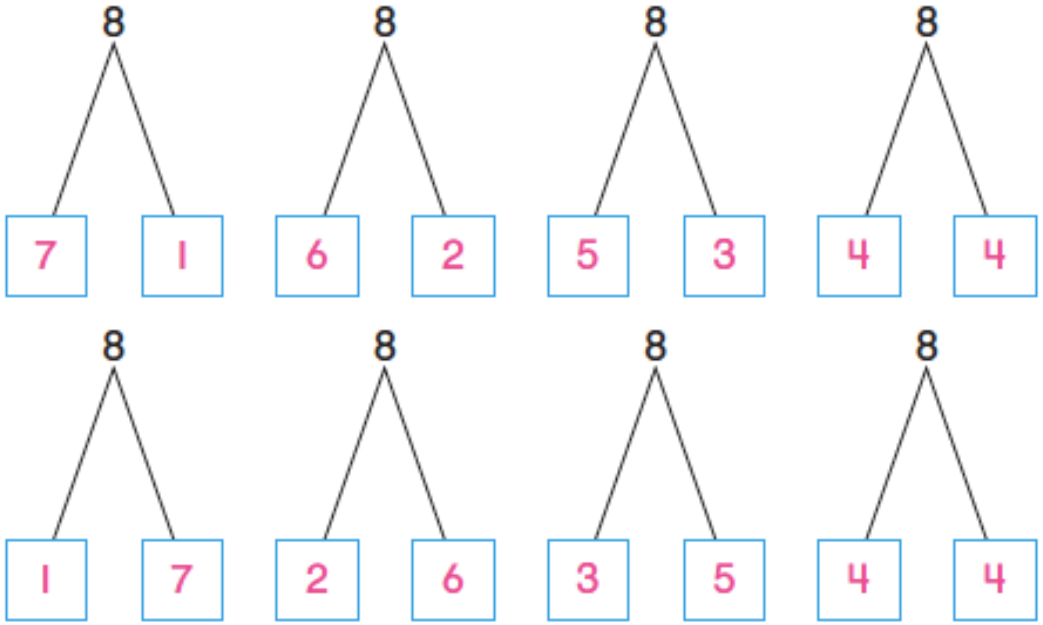
2.  6 + 2 and 2 + 6

3.  5 + 3 and 3 + 5

4.  4 + 4 and 4 + 4

Write the partners and the switched partners.

5. 

6. 

Grade 1 Teen Problems

	Stigler, Lee, Stephenson			
	Japanese Grade 1 $n = 120$	Chinese Grade 1 $n = 120$	U.S. Grade 1 $n = 240$	*Math Expressions* Grade 1 $n = 90$
Addition 9 + 4 word problem: Some squirrels picked up 9 nuts yesterday and 4 nuts today. How many nuts do they have altogether?	88	76**	64***	90
Subtraction 15 – 9 word problem: There were 15 bunnies. 9 hopped away. How many bunnies were left?	66***	38***	30***	89
Totals ≤10: 5 + 4 and 9 – 1				98

Grade 1 Counting On Supports for Adding

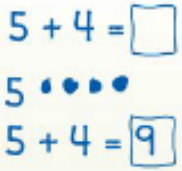
Children's Strategies

Compare Counting Strategies Children are introduced to comparing "counting all" to "counting on." *Counting on* differs from *counting all* in that the counting is abbreviated by counting on from the greater number. This is especially important when children start adding numbers with totals greater than 10.

Count On with Fingers Then children learn how to use their fingers to count on to find a total. The child is monitoring the known partner to decide when to stop counting.

I have 5.

Count On with Dots Children also learn how to draw dots to represent one addend. Then they start with one partner and count the dots to find the total.



	Result Unknown	Change Unknown	Start Unknown
Add To	*A* bunnies sat on the grass. *B* more bunnies hopped there. How many bunnies are on the grass now? A + B = ☐ **K**	*A* bunnies were sitting on the grass. Some more bunnies hopped there. Then there were *C* bunnies. How many bunnies hopped over to the first two? A + ☐ = C **1**	Some bunnies were sitting on the grass. *B* more bunnies hopped there. Then there were *C* bunnies. How many bunnies were on the grass before? ☐ + B = C **2**
Take From	*C* apples were on the table. I ate *B* apples. How many apples are on the table now? C – B = ☐ **K**	*C* apples were on the table. I ate some apples. Then there were *A* apples. How many apples did I eat? C – ☐ = A **1**	Some apples were on the table. I ate *B* apples. Then there were *A* apples. How many apples were on the table before? ☐ – B = A **2**

	Total Unknown	Both Addends Unknown[1]	Addend Unknown[2]
Put Together /Take Apart	*A* red apples and *B* green apples are on the table. How many apples are on the table? A + B = ☐ **K**	Grandma has *C* flowers. How many can she put in her red vase and how many in her blue vase? C = ☐ + ☐ **K**	*C* apples are on the table. *A* are red and the rest are green. How many apples are green? A + ☐ = C C – A = ☐ **1**

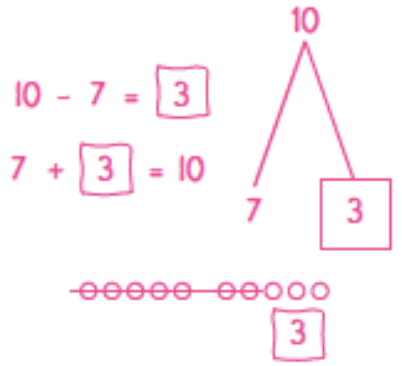
	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare	*"How many more?" version.* Lucy has *A* apples. Julie has *C* apples. How many more apples does Julie have than Lucy? **1** *"How many fewer?" version.* Lucy has *A* apples. Julie has *C* apples. How many fewer apples does Lucy have than Julie? A + ☐ = C C – A = ☐ **1**	*"More" version suggests operation.* Julie has *B* more apples than Lucy. Lucy has *A* apples. How many apples does Julie have? **1** *"Fewer" version suggests wrong operation.* Lucy has *B* fewer apples than Julie. Lucy has *A* apples. How many apples does Julie have? A + B = ☐ **2**	*"Fewer" version suggests operation.* Lucy has *B* fewer apples than Julie. Julie has *C* apples. How many apples does Lucy have? **1** *"More" version suggests wrong operation.* Julie has *B* more apples than Lucy. Julie has *C* apples. How many apples does Lucy have? C – B = ☐ ☐ + B = C **2**

Grade 1 Relating Subtraction Representations

Solve and discuss.

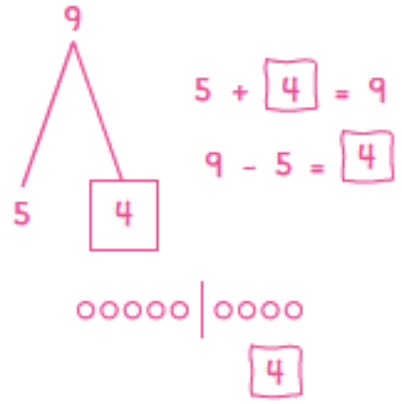
1. We see 10 dogs.
   7 run away.
   How many are left?

   3 dogs
   label



2. We see 9 dogs.
   5 are not barking.
   The rest are barking.
   How many are barking?

   4 dogs
   label



3. **Discuss** How are the methods you used to solve the problems alike and different?

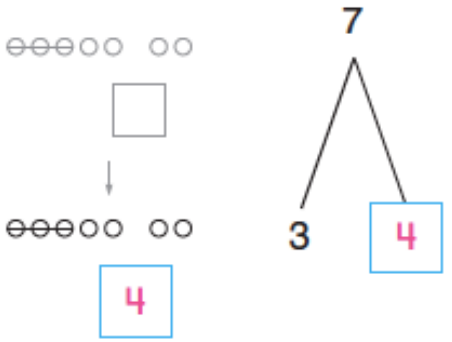
Grade 1
Varying the Unknown for Subtraction

Solve and discuss.

4. There are 7 cats.
3 cats walk away.
How many cats are left?

$7 - 3 = \boxed{4}$

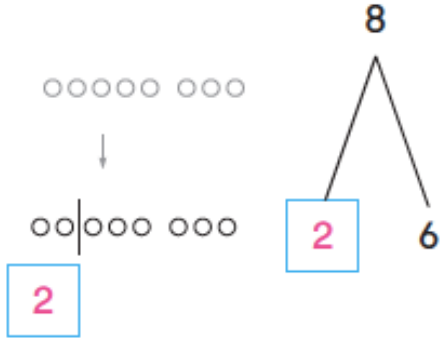
$3 + \boxed{4} = 7$



5. There are 8 cats.
Some cats walk away.
There are 6 cats left.
How many cats walk away?

$8 - \boxed{2} = 6$

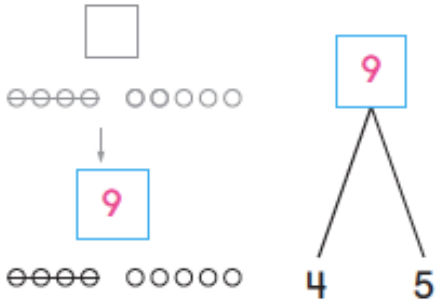
$6 + \boxed{2} = 8$



6. There are some cats.
4 cats walk away.
Now there are 5 cats.
How many cats are there at the start?

$\boxed{9} - 4 = 5$

$5 + 4 = \boxed{9}$



Grade 1 Unknown Addend Performance

Grade 1 Answer, Strategy, and Error Results for Easy and Intermediate Non-Compare Word Problems

Problem Difficulty	Correct Answer	Correct Strategy/ Incorrect Answer	Incorrect Strategy/ Incorrect Answer
Totals ≤ 10 (from Unit 3 Test, given in December)			
Easy Problems	93%	4%	3%
Intermediate Problems	71%	18%	11%
Totals 11 to 18 (from Unit 5 Test, given in March)			
Easy Problems	83%	9%	8%
Intermediate Problems	83%	6%	11%

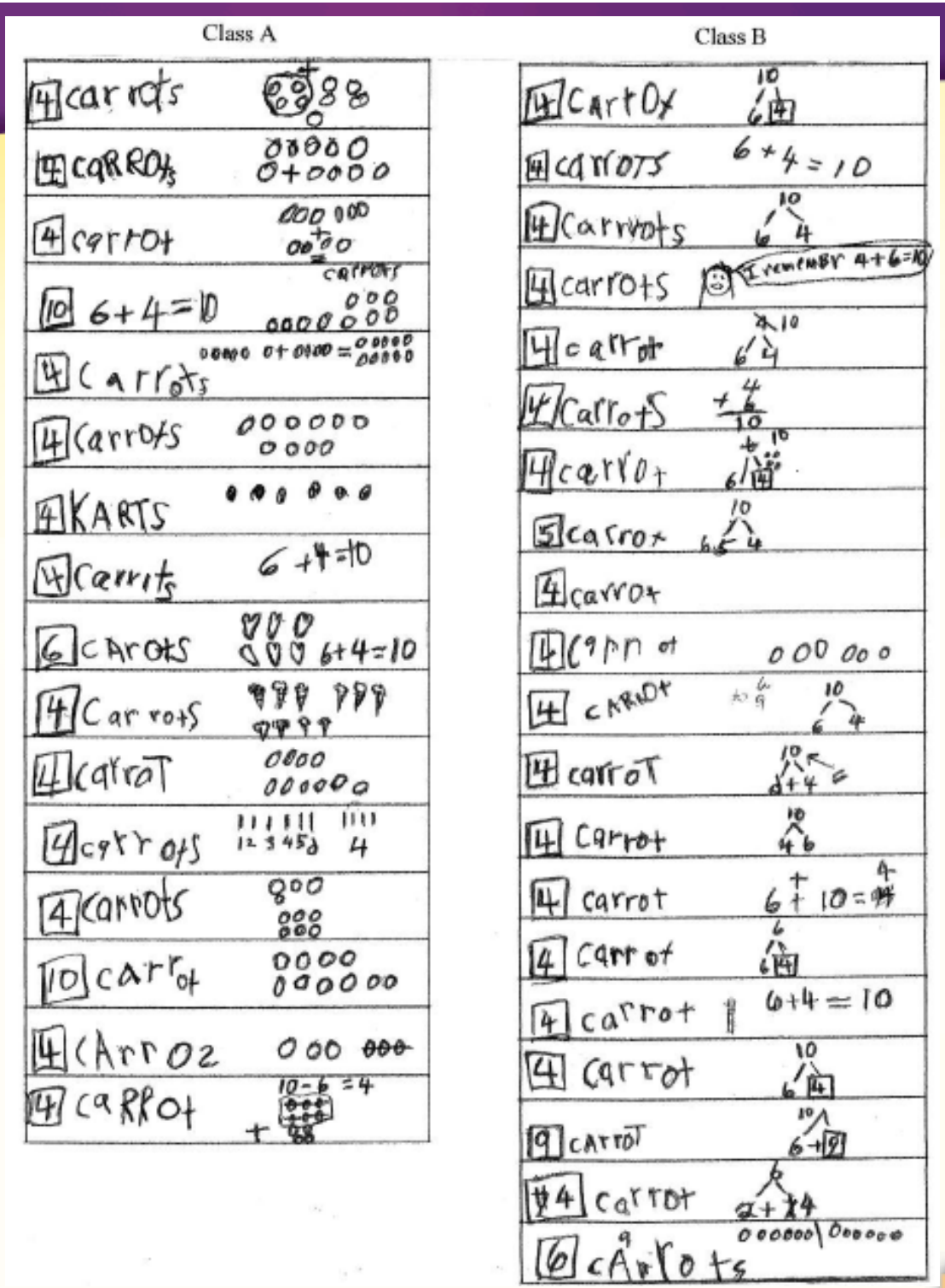
PT: Addend Unknown

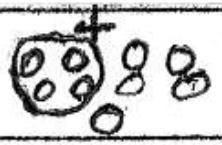
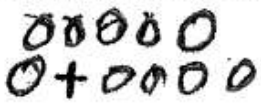
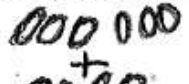
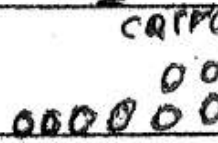
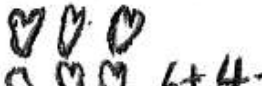
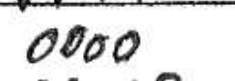
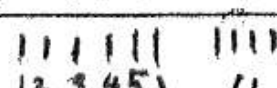
Rosa picked 6 carrots.

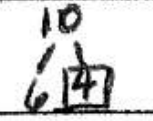
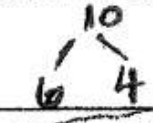
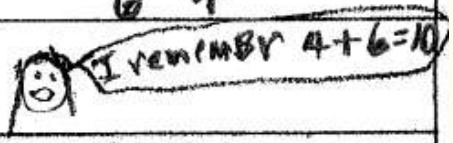
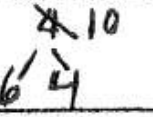
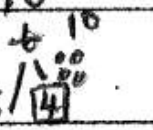
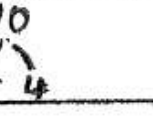
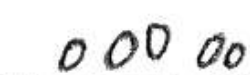
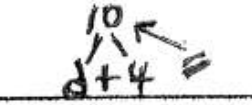
Her sister picked some too.

Together they picked 10 carrots.

How many did Rosa's sister pick?



4 carrots	
4 carrots	
4 carrot	
10 6+4=10	carrots 
4 carrots	
4 carrots	
4 KARTS	
4 carrits	6+4=10
6 CAROTS	 6+4=10
4 Carrots	
4 carrot	
4 carrots	

4 Carrot	
4 carrots	6+4=10
4 carrots	
4 carrots	 I remember 4+6=10
4 carrot	
4 Carrots	
4 carrot	
5 carrot	
4 carrot	
4 Carrot	
4 CARROT	
4 carrot	
4 Carrot	

Grade 2 classes had a quick overview with eliciting from students of the approaches in Grade 1 for those who were not present in Grade 1.

Grade 2 Performance

Grade 2 Answer, Strategy, and Error Results for Easy, Intermediate, Compare, and Difficult Word Problem Types with Totals 11 to 18

Problem Difficulty	Correct Answer	Correct Strategy/ Incorrect Answer	Incorrect Strategy/ Incorrect Answer
Easy Problems	96%	0%	4%
Intermediate Problems	89%	5%	5%
Compare Problems	82%	8%	10%
Difficult Problems	77%	4%	20%

Take From: Change Unknown

Jenna has 11 goldfish.

She gives some to her friend.

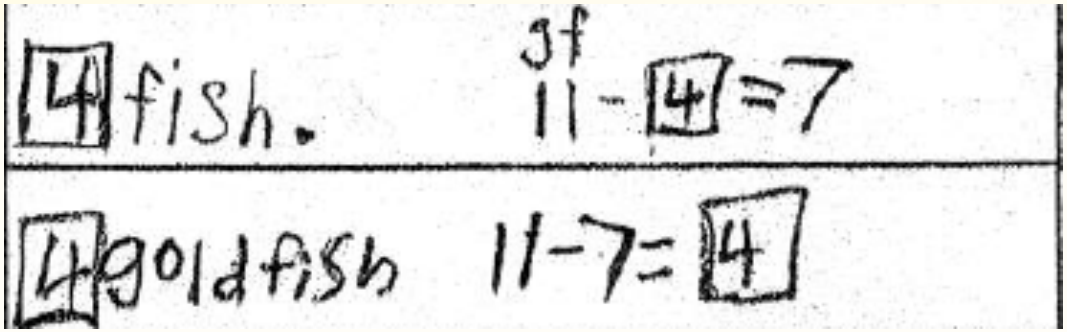
Now she only has 7 goldfish.

How many goldfish did she give to her friend?

[4] goldfish She gives to her friend	J f J 11 - [4] = 7
[4] goldfish	gf 11 - [4] = 7
[4] goldfish	11 - 4 = 7 4 goldfish
[4] goldfish	11 - [4] = 7 goldfish goldfish
[4] goldfish	11 - 7 = [4]
[3] goldfish	
[4] goldfish	11 - [4] =
[4] goldfish	11 - [4] = 7
[4] goldfish	11 - [4] = 7
[3] are gone	11 - [3] = 7
[4] fish.	gf 11 - [4] = 7
[4] goldfish	11 - 7 = [4]

[4] goldfish	11 in all 4 + 7 9 left
[4] goldfishs	11 - [4] = 7 J g J gave
[4] goldfish	11 - [4] = 7 J gave n
[4] goldfish	7 + [4] = 11
[4] goldfish	11 - [4] = 7 g R Mow
[4] goldfish	
[4] goldfish	gave | now
[4] goldfish	11 - [4] = 7 11
[4] goldfish	11 - 7 = 4
[4] goldfish	11 - [4] = 7 9 left
[4] goldfish	11 - [4] = 7
[4] goldfish	11 - 7 = [4] 7 + [4] = 11

Situation Equation



Solution Equation

Take From: Start Unknown

Joey had a bag of peanuts.

He gave 8 peanuts to his friend.

Then he had 7 left.

How many peanuts were in the bag?

15 peanuts J 1 bag 8+7=15

15 peanuts J 1111111 f 11111111

15 peanuts J 00000000 00000000

15 peanuts J 00000000 F 000000000

15 peanuts

15 Peanuts

15 peanuts J 00000000 F 00000000

15 peanuts 00000000/00000

15 Peanuts J f

15 peanuts

14 Peanuts J

15 Peanuts f J

15 peanuts J

15 peanuts 15 in all 8+7 9 left

15 peanuts 8 more add 7 8+7=15

15 peanuts 8+7=15 gave away in all

15 peanuts

15 peanuts 15 -8=7

15 peanuts 15-8=7

15 peanuts 8p to f now he has 15 in his bag

15 peanuts 8+7=15 15 8 7

1 peanuts 8-7=1

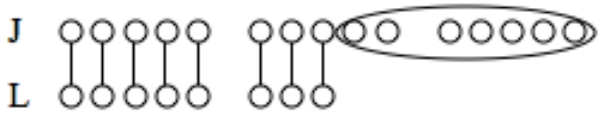
15 Peanuts 15-8=7 left

15 Peanuts 15-8=7

15 peanuts 8+7=15 15-7=8

In March Jana read 15 books. Lisa read 8 books.
How many fewer books did Lisa read than Jana?

Matching Drawing of Quantities



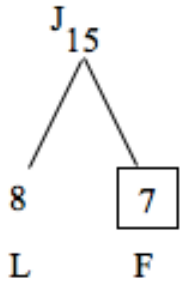
7 books

A Situation Equation

8 + 7 = 15
Lisa more Jana

7 books

Numerical Relationships Shown in Math Mountain



7 books

A Solution Equation

15 - 8 = 7
Jana Lisa fewer

7 books

Other Equations

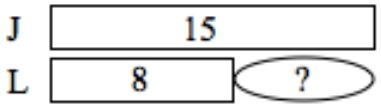
(7 was in the ☐)

☐ + 8 = 15

15 = ☐ + 8

15 – ☐ = 8

Comparison Bar Drawing of Quantities



7 books

Circle or Stick Drawings

Non-matching drawings were also made alone or with equations.

Figure 9. Grade 2 Solution Approaches to a **Compare: Difference Unknown** Problem

For larger numbers students will need to move from a situation equation to a solution equation or a computation.

Labeling the equation with situation content will be important.

See these NCTM books for more information about OA problem solving:

Focus in kindergarten:
Teaching with curriculum focal points

Focus in grade 1:
Teaching with curriculum focal points

Focus in grade 2:
Teaching with curriculum focal points

Visual models are central core ideas and practices in the CCSS and support reasoning and explaining.

The models can be simple math drawings
that students can make and use in their own ways
in problem solving and explaining of thinking.

We want classrooms to be using the mathematical practices in the Math Talk Community:
Students focus on math sense-making about math structure
using math drawings (visual models) to support math explaining.

Children Living in Poverty Can Solve CCSS OA Word Problems

Professor Karen C. Fuson and Steven T. Smith

Northwestern University

For more details about the CCSS-M and visual supports, please see the series of visual with audio Teaching Progressions I have made for various math domains. These can be found at karenfusonmath.com