

Karen C. Fuson

Research Shorter Publications for Teachers

- Fuson, K. C. (1991). Children's early counting: Saying the number-word sequence, counting objects, and understanding cardinality. In K. Durkin & B. Shire (Eds.), *Language and mathematical education* (pp. 27-39). Milton Keynes, GB: Open University Press.
- Fuson, K. C. & Kwon, Y. (1991). Chinese-based regular and European irregular systems of number words: The disadvantages for English-speaking children. In K. Durkin & B. Shire (Eds.), *Language and mathematical education* (pp. 211-226). Milton Keynes, GB: Open University Press.
- Fuson, K. C. (1992). Relationships between counting and cardinality from age 2 to age 8. In J. Bideaud, J. P. Fischer, & C. Meljac (Eds.), *Pathways to number* (pp. 127-149). Hillsdale, NJ: Lawrence Erlbaum Associates. [This is a book of invited chapters celebrating the fiftieth anniversary of the publication of Piaget's book on number. Published in French in *Les chemins du nombre* (pp. 159-179). Villeneuve d'Ascq, France: Presses Universitaires de Lille.]
- Fuson, K. C. & Kwon, Y. (1992). Learning addition and subtraction: Effects of number words and other cultural tools. In J. Bideaud & C. Meljac (Eds.), *Pathways to number* (283-306). Hillsdale, NJ: Lawrence Erlbaum Associates. [This is a book of invited chapters celebrating the fiftieth anniversary of the publication of Piaget's book on number. Published in French in *Les chemins du nombre* (pp. 351-374). Villeneuve d'Ascq, France: Presses Universitaires de Lille.]
- Fuson, K. C., & Burghardt, B. H. (1993). Group case studies of second graders inventing multidigit addition procedures for base-ten blocks and written marks. In J. R. Becker & B. J. Pence (Eds., 1993), *Proceedings of the Fifteenth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 240-246). The Center for Mathematics and Computer Science Education, San Jose State University, San Jose, CA.
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- Hiebert, J., Carpenter, T. P., Fennema, E., Fuson, K., Human, P., Murray, H., Olivier, A., & Wearne, D. (1996). Problem solving as a basis for reform in curriculum and instructions: The case of mathematics. *Educational Researcher*, Vol. 25, No. 4., pp. 12-21.
- Fuson, K. C., & Burghardt, B. H. (1997). Group case studies of second graders inventing multidigit subtraction methods. In J. A. Dossey, J. O. Swafford, M. Parmantie, & A.E. Dossey (Eds.), *Proceedings of the 19th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, 1*, (pp. 291-298). Columbus, OH: ERIC Clearinghouse for Science, Mathematics, and Environmental Education.
- Fuson, K. C., Lo Cicero, A., Hudson, K., & Smith, S. T. (1997). Snapshots across two years in the life of an urban Latino classroom. In Hiebert, J., Carpenter, T., Fennema, E., Fuson, K. C., Wearne, D., Murray, H., Olivier, A., Human, P., *Making sense: Teaching and learning mathematics with understanding* (pp. 129-159). Portsmouth, NH: Heinemann.
- Fuson, K. C., Sherin, B., & Smith, S. T. (1998). A Vygotskian action-research model for developing and assessing conceptual models and instructional materials inter-actively. In S. Berenson, K. Dawkins, M. Blanton, W. Coulombe, J. Kolb, K. Norwood, & L. Stiff (Eds.), *Proceedings of the Twentieth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, Vol. 2* (pp. 541-548). Raleigh, NC: North Carolina State University.
- Ron, M. P. (1998). My parent taught me this way: The European/Latino multidigit subtraction algorithm and confusions with a U.S. algorithm. In L. J. Morrow & M. J. Kenney (Eds.), *The teaching and learning of*

algorithms in school mathematics (pp. 115-119). Reston, VA: The National Council of Teachers of Mathematics.

- De La Cruz, Y. (1999). Reversing the trend: Latino families in real partnerships with schools. *Teaching Children Mathematics*, 5 (5), pp. 296-300.
- Lo Cicero, A., De La Cruz, Y., Fuson, K. C. (1999). Teaching and learning creatively with the Children's Math Worlds Curriculum: Using children's narratives and explanations to co-create understandings. *Teaching Children Mathematics*, 5 (9), 544-547.
- Ron, Pilar (1999). Spanish-English language issues in the mathematics classroom. In L. Ortiz-Franco, N. G. Hernandez, & Y. De La Cruz (Eds.) *Changing the Faces of Mathematics: Perspectives on Latinos* (pp. 23-33). Reston, Va.: National Council of Teachers of Mathematics.
- Fuson, K. C. & Lo Cicero, A. M. (2000). El Mercado in Latino primary math classrooms. In M. L. Fernandez (Ed.), *Proceedings of the Twenty-Second Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, Vol. 2* (p. 453). Columbus, OH: ERIC Clearinghouse for Science, Mathematics, and Environmental Education.
- Fuson, K. C., De La Cruz, Y., Smith, S., Lo Cicero, A., Hudson, K., Ron, P., & Steeby, R. (2000). Blending the best of the 20th century to achieve a Mathematics Equity Pedagogy in the 21st century. In M. J. Burke & F. R. Curcio (Eds.), *Learning mathematics for a new century* (pp. 197-212). Reston, VA: NCTM.
- Izsák, A. & Fuson, K. C. (2000). Students' understanding and use of multiple representations while learning two-digit multiplication. In M. L. Fernandez (Ed.), *Proceedings of the Twenty-Second Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, Vol. 2* (pp. 714-721). Columbus, OH: ERIC Clearinghouse for Science, Mathematics, and Environmental Education.
- Lee, Hf. (2000). Supports for learning multi-digit addition and subtraction: A study of Taiwanese second-grade low-math achievers. In M. L. Fernandez (Ed.), *Proceedings of the Twenty-Second Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, Vol. 2* (pp. 723-728). Columbus, OH: ERIC Clearinghouse for Science, Mathematics, and Environmental Education.
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- Fuson, K. C., Grandau, L., & Sugiyama, P. (2001). Achievable numerical understandings for all young children. *Teaching Children Mathematics*, 7(9), 522-526.
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- Kalchman, M. & Fuson, K. (2001). Conceptual understanding of functions: A tale of two schemas. In R. Speiser, C. S. Maher, & C. Walter (Eds., 2001), *Proceedings of the Twenty-Third Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, Vol. 1* (pp. 195-205). Columbus, OH: ERIC Clearinghouse for Science, Mathematics, and Environmental Education.
- Murata, A. & Fuson, K. (2001). Learning paths to 5- and 10-structured understanding of quantity: Addition and subtraction solution strategies of Japanese children. In R. Speiser, C. S. Maher, & C. Walter (Eds.), *Proceedings of the Twenty-Third Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, Vol. 2* (pp. 639-646). Columbus, OH: ERIC Clearinghouse for Science, Mathematics, and Environmental Education.

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- Fuson, K. and Kalchman, M. (2002). A length model of fractions puts multiplication of fractions in the learning zone of fifth graders. In D. L. Haury (Ed.), *Proceedings of the twenty-fourth annual meeting of North American chapter of the International Group of the Psychology of Mathematics Education* (pp. 1641 – 1649). Columbus, OH: ERIC Clearinghouse for Science, Mathematics, and Environmental Education.
- Fuson, K. C., Smith, S. T., & Lo Cicero, A. (2002). Supporting Latino first graders' ten-structured thinking in urban classrooms. In J. Sowder & B. Schapelle (Eds.), *Lessons Learned from Research* (pp. 155-162). Reston, VA: NCTM.
- Abrahamson, D. (2003). Text talk, body talk, table talk: A design of ratio and proportion as classroom parallel events. *Proceedings of the 27th annual meeting of the International Group for the Psychology of Mathematics Education, Honolulu, Hawaii, Vol. 2* (pp. 1-8). Columbus, OH: Eric Clearinghouse for Science, Mathematics, and Environmental Education.
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- Fuson, K. (2012). The common core mathematics standards as supports for learning and teaching early and elementary school. In J. S. Carlson & J. R. Levin (Eds.), *Instructional strategies for improving student learning: Focus on early math and reading* (pp. 177-186). Vol. 3 in Psychological perspectives on contemporary educational issues. Charlotte, NC: Information Age Publishing.
- Fuson, K. C. (2014). CCSS Mathematical Practice 4: Model with mathematics. *Colorado Mathematics Teacher, Spring*, 10-21.
- Fuson, K.C., Clements, D.H., & Sarama, J. (2015). Making early math education work for all children. *Phi Delta Kappan, 97 (3 November)*, 63-68. Doi:10.1177/0031721715614831
- Hufferd-Ackles, K., Fuson, K. C., & Sherin, M. G. (2015). Describing levels and components of a Math-Talk Learning Community. In E. A. Silver & P. A. Kenney (Eds.), *More lessons learned from research: Volume 1: Useful and usable research related to core mathematical practices* (pp. 125-134). Reston, VA: NCTM.
- Izsák, A., Tellema, E., & Tunç -Pekkan, Z. (2016). Partitioning and iterating when teaching and learning fraction addition on number lines. In E. A. Silver & P. A. Kenney (Eds.), *More lessons learned from research: Volume 2: Useful and usable research related to core mathematical practices* (pp. 199-205). Reston, VA: NCTM.
- Murata, A. & Fuson, K. C. (2016). Class learning zone and class learning paths: Responsive teaching in first-grade mathematics. In E. A. Silver & P. A. Kenney (Eds.), *More lessons learned from research: Volume 2: Useful and usable research related to core mathematical practices* (pp. 70-91). Reston, VA: NCTM.

Sherin, B. Fuson, K. C. (2016). Multiplication and division methods in the context of the Common Core State Standards. In E. A. Silver & P. A. Kenney (Eds.), *More lessons learned from research: Volume 2: Useful and usable research related to core mathematical practices* (pp. 101-108). Reston, VA: NCTM.

Longer Research Publications

Fuson, K. C., Richards, J., & Briars, D. J. (1982). The acquisition and elaboration of the number word sequence. In C. Brainerd (Ed.), *Progress in cognitive development: children's logical and mathematical cognition*, Vol. 1 (pp. 33-92). New York: Springer-Verlag.
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Fuson, K. C. (1986). Teaching children to subtract by counting up. *Journal for Research in Mathematics Education*, 17, 172-189. This paper was chosen as the best research article of 1986 by the Research Advisory Council of the National Council of Teachers of Mathematics.

Stigler, J., Fuson, K. C., Ham, M., & Kim, M. S. (1986). An analysis of addition and subtraction word problems in Soviet and American elementary textbooks. *Cognition and Instruction*, 3, 153-171.

Fuson, K. C. (1988). *Children's counting and concepts of number*. New York: Springer-Verlag. [Table of Contents and Chapter 1: Introduction and overview of different uses of number words]

Fuson, K. C. (1988). *Children's counting and concepts of number*. New York: Springer-Verlag. [Chapter 11: An overview of changes in children's number word concepts from age 2 through 8]

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Fuson, K. C., & Willis, G. B. (1988). Subtracting by counting up: More evidence. *Journal for Research in Mathematics Education*, 19, 402-420.

Fuson, K. C. (1990). Issues in place-value and multidigit addition and subtraction learning. *Journal for Research in Mathematics Education*, 21, 273-280.

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Fuson, K. C. & Kwon, Y. (1992). Korean children's single-digit addition and subtraction: Numbers structured by ten. *Journal for Research in Mathematics Education*, 23, 148-165.

- Fuson, K. C., Fraivillig, J. L., & Burghardt, B. H. (1992). Relationships children construct among English number words, multiunit base-ten blocks, and written multidigit addition. In J. Campbell (Ed.), *Advances in psychology: The nature and origins of mathematical skills* (39-112). North Holland, Elsevier Science.
- Fuson, K. C., & Smith, S. T. (1995). Complexities in learning two-digit subtraction: A case study of tutored learning. *Mathematical Cognition, 1*, 165-213.
- Fuson, K. C., Zecker, L. B., Lo Cicero, A. M., Ron, P. (1995, April). *El Mercado in Latino primary classrooms: A fruitful narrative theme for the development of children's conceptual mathematics*. Paper presented at the Annual Meeting of the American Educational Research Association, San Francisco, CA.
- Fuson, K. C., Carroll, W. M., & Landis, J. (1996). Levels in conceptualizing and solving addition/subtraction compare word problems. *Cognition and Instruction, 14*(3), 345-371.
- Fuson, K. C., Wearne, D., Hiebert, J., Human, P., Murray, H., Olivier, A., Carpenter, T., & Fennema, E. (1997). Children's conceptual structures for multidigit numbers at work in addition and subtraction. *Journal for Research in Mathematics Education, 28*, 130-162.
- Fuson, K. C., Smith, S. T., & Lo Cicero, A. (1997). Supporting Latino first graders' ten-structured thinking in urban classrooms. *Journal for Research in Mathematics Education, 28*, 738-766.
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